Ellipses: Standard Form & Key Parts

Pre-Calculus / Analytic Geometry Worksheet · Grade 10–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Identify the center, foci, and lengths of axes of an ellipse from its standard equation
- Write the standard equation of an ellipse given its foci and major axis length
- Convert a general second-degree equation to the standard form of an ellipse by completing the square

Problems

1. Identify the center of the ellipse with the equation shown.

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

2. For the ellipse shown, determine the values of a squared and b squared, then state which axis is the major axis.

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

3. Find the length of the major axis and the length of the minor axis for the ellipse shown.

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

4. Find the coordinates of the foci of the ellipse shown.

$$\frac{x^2}{49} + \frac{y^2}{24} = 1$$

5. An ellipse has its center at (3, 2), a horizontal major axis, a value of a equal to 5, and a value of b equal to 3. Write the standard equation of this ellipse.

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6. An ellipse has foci at (0, 1) and (4, 1) and a major axis of length 6. Find the center, the value of c, the value of a, b squared, and write the standard equation.

7. An ellipse has foci at (2, –1) and (2, 5) and a major axis of length 10. Find the standard equation of the ellipse.

8. Convert the general equation shown to standard form by completing the square, then identify the center.

$$x^2 + 4y^2 + 6x - 8y + 9 = 0$$

9. Convert the general equation shown to standard form, identify the center and foci, and state the lengths of both axes.

$$4x^2 + 9y^2 - 16x + 18y - 11 = 0$$

10. An ellipse in standard form has center at (–1, 3), one focus at (–1, 3 + 2√3), and one vertex of the major axis at (–1, 7). Write the standard equation and find the coordinates of both foci and all four vertices.

Ellipses: Standard Form & Key Parts — Answer Key

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Answer Key

1. Answer: Center: (0, 0)

- The equation is already in standard form with $h = 0$ and $k = 0$.
- Center = $(h, k) = (0, 0)$.

2. Answer: $a^2 = 25$, $b^2 = 4$; major axis is along the x-axis

- The denominator under x^2 is 25 and under y^2 is 4.
- Since $25 > 4$, $a^2 = 25$ and $b^2 = 4$.
- The larger denominator is under x^2, so the major axis is horizontal (along the x-axis).

3. Answer: Major axis length = 12; Minor axis length = 8

- $a^2 = 36$, so $a = 6$. The major axis has length $2a = 12$.
- $b^2 = 16$, so $b = 4$. The minor axis has length $2b = 8$.

4. Answer: Foci: (5, 0) and (–5, 0)

- $a^2 = 49$ (larger, under x^2) and $b^2 = 24$.
- Use $c^2 = a^2 - b^2 = 49 - 24 = 25$, so $c = 5$.
- Since the major axis is horizontal, foci are at $(\pm 5, 0)$.

5. Answer: $(x - 3)^2 / 25 + (y - 2)^2 / 9 = 1$

- Center $(h, k) = (3, 2)$, $a = 5$ so $a^2 = 25$, $b = 3$ so $b^2 = 9$.
- Major axis is horizontal, so a^2 goes under $(x - h)^2$.
- Standard form: $(x - 3)^2 / 25 + (y - 2)^2 / 9 = 1$.

6. Answer: Center (2,1); $c=2$; $a=3$; $b^2=5$; equation: $(x-2)^2/9 + (y-1)^2/5 = 1$

- Center = midpoint of foci = $((0+4)/2, (1+1)/2) = (2, 1)$.
- c = distance from center to a focus = 2.
- Major axis = $2a = 6$, so $a = 3$ and $a^2 = 9$.
- $b^2 = a^2 - c^2 = 9 - 4 = 5$.
- Foci share the same y-coordinate, so the major axis is horizontal.
- Standard equation: $(x - 2)^2 / 9 + (y - 1)^2 / 5 = 1$.

7. Answer: $(x-2)^2/16 + (y-2)^2/25 = 1$

- Center = midpoint = $((2+2)/2, (-1+5)/2) = (2, 2)$.
- c = distance from center (2,2) to focus (2,5) = 3.
- Major axis = $2a = 10$, so $a = 5$ and $a^2 = 25$.
- $b^2 = a^2 - c^2 = 25 - 9 = 16$.
- Foci share the same x-coordinate, so the major axis is vertical; a^2 goes under $(y - k)^2$.

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- Standard equation: $(x - 2)^2 / 16 + (y - 2)^2 / 25 = 1$.

8. Answer: $(x+3)^2/4 + (y-1)^2/1 = 1$; Center: $(-3, 1)$

- Group: $(x^2 + 6x) + 4(y^2 - 2y) = -9$.
- Complete the square for x: $(x^2 + 6x + 9)$ adds 9 to the left and right.
- Complete the square for y inside the factor of 4: $(y^2 - 2y + 1)$ adds $4 \cdot 1 = 4$ to the right.
- Equation becomes: $(x + 3)^2 + 4(y - 1)^2 = -9 + 9 + 4 = 4$.
- Divide both sides by 4: $(x + 3)^2/4 + (y - 1)^2/1 = 1$.
- Center: $(-3, 1)$.

9. Answer: $(x-2)^2/9 + (y+1)^2/4 = 1$; Center $(2,-1)$; $c=\sqrt{5}$; Major axis length 6; Minor axis length 4

- Group: $4(x^2 - 4x) + 9(y^2 + 2y) = 11$.
- Complete x: $4(x^2 - 4x + 4)$ adds $4 \cdot 4 = 16$; complete y: $9(y^2 + 2y + 1)$ adds $9 \cdot 1 = 9$.
- Equation: $4(x - 2)^2 + 9(y + 1)^2 = 11 + 16 + 9 = 36$.
- Divide by 36: $(x - 2)^2/9 + (y + 1)^2/4 = 1$.
- Center: $(2, -1)$; $a^2 = 9$, $b^2 = 4$.
- $c^2 = 9 - 4 = 5$, so $c = \sqrt{5}$. Foci at $(2 \pm \sqrt{5}, -1)$.
- Major axis length = $2a = 6$; Minor axis length = $2b = 4$.

10. Answer: $(x+1)^2/4 + (y-3)^2/16 = 1$; Foci: $(-1, 3\pm 2\sqrt{3})$; Vertices: $(-1,7),(-1,-1),(1,3),(-3,3)$

- Center $(h, k) = (-1, 3)$. The vertex at $(-1, 7)$ is directly above the center, so the major axis is vertical.
- a = distance from center to vertex = $|7 - 3| = 4$, so $a^2 = 16$.
- c = distance from center to focus = $2\sqrt{3}$, so $c^2 = 12$.
- $b^2 = a^2 - c^2 = 16 - 12 = 4$.
- Standard equation: $(x + 1)^2/4 + (y - 3)^2/16 = 1$.
- Foci (vertical major axis): $(-1, 3 + 2\sqrt{3})$ and $(-1, 3 - 2\sqrt{3})$.
- Major-axis vertices: $(-1, 3 + 4) = (-1, 7)$ and $(-1, 3 - 4) = (-1, -1)$.
- Minor-axis vertices: $(-1 + 2, 3) = (1, 3)$ and $(-1 - 2, 3) = (-3, 3)$.