Analyzing Ellipses: Standard Form and Graphing

Precalculus / Algebra 2 Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Find the center, foci, and vertices of an ellipse given its foci and major axis length
- Derive the standard equation of an ellipse using the values of a, b, and the center
- Graph an ellipse by identifying its key features including center, vertices, and foci

Problems

1. An ellipse has foci at (0, 1) and (4, 1). Find the center of the ellipse.

$$F_1 = (0, 1), \quad F_2 = (4, 1)$$

2. An ellipse has foci at (0, 1) and (4, 1). Find the value of c, the distance from the center to each focus.

$$F_1 = (0, 1), \quad F_2 = (4, 1), \quad \text{Center} = (2, 1)$$

3. An ellipse has a major axis of length 6. Find the value of a.

$$2a = 6$$

4. Using the values c equals 2 and a equals 3, find the value of b squared.

$$c^2 = a^2 - b^2$$

5. Write the standard equation of the ellipse with center (2, 1), a squared equals 9, and b squared equals 5, where the major axis is horizontal.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

6. Find the coordinates of the two horizontal vertices of the ellipse below.

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$$\frac{(x-2)^2}{9}+\frac{(y-1)^2}{5}=1$$

7. Find the coordinates of the two vertical co-vertices of the ellipse below.

$$\frac{(x-2)^2}{9}+\frac{(y-1)^2}{5}=1$$

8. An ellipse has foci at (2, 0) and (2, 4) and a major axis of length 10. Find the center, the values of a, c, and b squared.

$$F_1=(2,0),\quad F_2=(2,4),\quad \text{major axis}=10$$

9. Write the standard equation of the ellipse with foci at (2, 0) and (2, 4) and major axis of length 10. The major axis is vertical.

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

10. Given the standard equation of an ellipse below, identify the center, foci, and vertices, then describe the orientation of the major axis.

$$\frac{(x+3)^2}{16}+\frac{(y-5)^2}{25}=1$$

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Analyzing Ellipses: Standard Form and Graphing — Answer Key

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Answer Key

1. Answer: (2, 1)
- The center is the midpoint of the two foci.
- Center = \left(\frac{0+4}{2}, \frac{1+1}{2}\right) = (2, 1)

2. Answer: c = 2
- c is the distance from the center (2, 1) to either focus.
- c = |2 - 0| = 2

3. Answer: a = 3
- The major axis length equals 2a.
- 2a = 6 \Rightarrow a = 3

4. Answer: b squared = 5
- Substitute c = 2 and a = 3 into the formula.
- 4 = 9 - b^2 \Rightarrow b^2 = 9 - 4 = 5

5. Answer: (x-2)^2 / 9 + (y-1)^2 / 5 = 1
- Substitute h = 2, k = 1, a^2 = 9, b^2 = 5.
- \frac{(x-2)^2}{9} + \frac{(y-1)^2}{5} = 1

6. Answer: (-1, 1) and (5, 1)
- The horizontal vertices are found by moving a = 3 units left and right from the center (2, 1).
- Left vertex: (2 - 3, 1) = (-1, 1); Right vertex: (2 + 3, 1) = (5, 1)

7. Answer: (2, 1 + sqrt(5)) and (2, 1 - sqrt(5))
- The vertical co-vertices are found by moving b = \sqrt{5} \approx 2.24 units up and down from center (2, 1).
- Top: (2, 1 + \sqrt{5}); Bottom: (2, 1 - \sqrt{5})

8. Answer: Center = (2, 2); a = 5; c = 2; b squared = 21
- Center = midpoint of foci = \left(\frac{2+2}{2}, \frac{0+4}{2}\right) = (2, 2)
- 2a = 10 \Rightarrow a = 5; \quad c = |2 - 0| = 2
- b^2 = a^2 - c^2 = 25 - 4 = 21

9. Answer: (x-2)^2 / 21 + (y-2)^2 / 25 = 1
- Center = (2, 2), a^2 = 25, b^2 = 21. Since the major axis is vertical, a^2 goes under the y term.
- \frac{(x-2)^2}{21} + \frac{(y-2)^2}{25} = 1

10. Answer: Center (-3, 5); major axis vertical; vertices (-3, 0) and (-3, 10); foci at (-3, 5 ± 3)

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- Center: h = -3, k = 5. Since 25 > 16, the major axis is vertical, so a^2 = 25 and b^2 = 16.
- a = 5, b = 4. Vertices: (-3, 5+5) = (-3, 10) and (-3, 5-5) = (-3, 0).
- c^2 = a^2 - b^2 = 25 - 16 = 9 \Rightarrow c = 3. Foci: (-3, 5+3) = (-3, 8) and (-3, 5-3) = (-3, 2).

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