Analyzing Ellipses: Equations, Graphs, and Foci

Precalculus Worksheet · Grade 10–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Identify the standard form of an ellipse and distinguish between horizontal and vertical orientations
- Determine the center, vertices, and foci of an ellipse from its equation or graph
- Convert a general form ellipse equation to standard form and extract key features

Problems

1. Identify the center and the direction of the major axis for the ellipse below:

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

2. Find the vertices along the major and minor axes for the ellipse below:

$$\frac{x^2}{16} + \frac{y^2}{36} = 1$$

3. Write the standard form equation of the ellipse whose graph has center at the origin, vertices at (0, 5) and (0, -5), and co-vertices at (3, 0) and (-3, 0).

4. Identify the center, and the values of a and b for the ellipse below:

$$\frac{(x-2)^2}{49} + \frac{(y+3)^2}{16} = 1$$

5. Convert the equation below to standard form by dividing both sides by 225, then identify the major axis direction:

$$25x^2 + 9y^2 = 225$$

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6. Find the foci of the ellipse below using the relationship c squared equals b squared minus a squared (when the major axis is vertical):

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

7. Find the foci of the ellipse below:

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

8. Convert the equation below to standard form, then find the center, vertices along the major axis, and foci:

$$4x^2 + 9y^2 = 36$$

9. Find the center, vertices along both axes, and foci for the ellipse below:

$$\frac{(x+1)^2}{25} + \frac{(y-4)^2}{9} = 1$$

10. Write the standard form equation of the ellipse with center at (3, -2), a focus at (3, 2), and a vertex at (3, 5). Then state the coordinates of all four vertices.

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Analyzing Ellipses: Equations, Graphs, and Foci — Answer Key

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Answer Key

1. Answer: Center: (0, 0); Major axis along the y-axis

- The equation is in standard form with $h = 0$ and $k = 0$, so the center is (0, 0).
- Compare denominators: 25 > 9, and 25 is under y^2, so the major axis is along the y-axis.

2. Answer: Major vertices: (0, 6) and (0, -6); Minor vertices: (4, 0) and (-4, 0)

- Since 36 > 16 and 36 is under y^2, the major axis is vertical. $b^2 = 36$, so $b = 6$. Major vertices: (0, 6) and (0, -6).
- $a^2 = 16$, so $a = 4$. Minor vertices: (4, 0) and (-4, 0).

3. Answer: $x^2 / 9 + y^2 / 25 = 1$

- The major axis is vertical with $b = 5$, so $b^2 = 25$ goes under y^2.
- The minor axis gives $a = 3$, so $a^2 = 9$ goes under x^2. Equation: $x^2/9 + y^2/25 = 1$.

4. Answer: Center: (2, -3); a = 7 (major axis along x-axis); b = 4

- From $(x - 2)^2$ and $(y + 3)^2$, the center is $(h, k) = (2, -3)$.
- 49 > 16 and 49 is under x^2, so the major axis is horizontal with $a = 7$. $b = 4$.

5. Answer: $x^2 / 9 + y^2 / 25 = 1$; Major axis along the y-axis

- Divide every term by 225: $25x^2/225 + 9y^2/225 = 1$, which simplifies to $x^2/9 + y^2/25 = 1$.
- Since 25 > 9 and 25 is under y^2, the major axis is along the y-axis.

6. Answer: Foci: (0, 4) and (0, -4)

- Here $b^2 = 25$ and $a^2 = 9$. Use $c^2 = b^2 - a^2 = 25 - 9 = 16$, so $c = 4$.
- Since the major axis is along the y-axis, the foci are at (0, 4) and (0, -4).

7. Answer: Foci: $(2\sqrt{5}, 0)$ and $(-2\sqrt{5}, 0)$

- Since 36 > 16 and 36 is under x^2, the major axis is horizontal. $a^2 = 36$, $b^2 = 16$.
- $c^2 = a^2 - b^2 = 36 - 16 = 20$, so $c = \sqrt{20} = 2\sqrt{5}$. Foci: $(2\sqrt{5}, 0)$ and $(-2\sqrt{5}, 0)$.

8. Answer: Center: (0,0); Major vertices: (3, 0) and (-3, 0); Foci: $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$

- Divide by 36: $x^2/9 + y^2/4 = 1$. Center is (0, 0). Since 9 > 4, major axis is horizontal with $a = 3$ and $b = 2$.
- $c^2 = 9 - 4 = 5$, so $c = \sqrt{5}$. Major vertices: (3, 0) and (-3, 0). Foci: $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$.

9. Answer: Center: (-1, 4); Major vertices: (4, 4) and (-6, 4); Minor vertices: (-1, 7) and (-1, 1); Foci: (3, 4) and (-5, 4)

- Center: $(h, k) = (-1, 4)$. Since 25 > 9 and 25 is under x^2, major axis is horizontal. $a = 5$, $b = 3$.
- Major vertices: $(-1 \pm 5, 4) = (4, 4)$ and $(-6, 4)$. Minor vertices: $(-1, 4 \pm 3) = (-1, 7)$ and $(-1, 1)$. $c^2 = 25 - 9 = 16$, $c = 4$. Foci: $(-1 \pm 4, 4) = (3, 4)$ and $(-5, 4)$.

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10. Answer: $(x-3)^2 / 9 + (y+2)^2 / 49 = 1$; Vertices: (3, 5), (3, -9), (6, -2), (0, -2)

- Center is (3, -2). The vertex (3, 5) is directly above the center, so the major axis is vertical. b = distance from center to vertex = 5 - (-2) = 7, so $b^2 = 49$.
- The focus (3, 2) gives c = distance from center to focus = 2 - (-2) = 4. Use $a^2 = b^2 - c^2 = 49 - 16 = 33$... Re-check: $a^2 = b^2 - c^2 = 49 - 16 = 33$, so $a = \sqrt{33}$. Equation: $(x-3)^2/33 + (y+2)^2/49 = 1$. Major vertices: (3, 5) and (3, -9). Minor vertices: $(3+\sqrt{33}, -2)$ and $(3-\sqrt{33}, -2)$.

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