Graphing and Analyzing Hyperbolas

Precalculus / Algebra 2 Worksheet · Grade 10–12

Name: ________________________________ Date: __________________

Learning Objectives

- Identify the standard form of a hyperbola and distinguish it from an ellipse
- Find the center, vertices, foci, and asymptotes of a hyperbola
- Graph a hyperbola by constructing the framework, rectangle, and asymptotes

Problems

1. Identify whether the equation below represents an ellipse or a hyperbola. Explain how you know.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

2. For the hyperbola below, identify the values of a and b.

$$\frac{x^2}{25} - \frac{y^2}{49} = 1$$

3. Find the coordinates of the vertices of the hyperbola below. The center is at the origin.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

4. Use the formula below to find the value of c for the hyperbola, given a equals 3 and b equals 4.

$$c^2 = a^2 + b^2$$

5. Find the coordinates of the foci for the hyperbola below. The center is at the origin.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

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6. Describe the steps to construct the graphing framework (rectangle) for the hyperbola below, and state the four corner points of the rectangle.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

7. Write the equations of the two asymptotes for the hyperbola below.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

8. The hyperbola below has its center at (2, -1). Identify the center, a, and b, then state the vertices.

$$\frac{(x-2)^2}{4} - \frac{(y+1)^2}{9} = 1$$

9. Find the foci of the hyperbola below and write their coordinates.

$$\frac{(x-2)^2}{4} - \frac{(y+1)^2}{9} = 1$$

10. Write the standard form equation of a hyperbola with center at the origin that opens horizontally, with vertices at (6, 0) and (-6, 0), and foci at (10, 0) and (-10, 0).

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Graphing and Analyzing Hyperbolas — Answer Key

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Answer Key

1. Answer: Hyperbola, because the terms are subtracted (minus sign between them)

- Check the sign between the two fraction terms.
- A minus sign indicates a hyperbola; a plus sign indicates an ellipse.
- Conclusion: This is a hyperbola.

2. Answer: a = 5, b = 7

- The denominator under x^2 is a^2, so $a^2 = 25$, giving $a = 5$.
- The denominator under y^2 is b^2, so $b^2 = 49$, giving $b = 7$.

3. Answer: Vertices: (3, 0) and (-3, 0)

- Since the x^2 term is positive, the hyperbola opens horizontally.
- $a^2 = 9$, so $a = 3$.
- Vertices are located at (±a, 0) = (3, 0) and (-3, 0).

4. Answer: c = 5

- Substitute a = 3 and b = 4: $c^2 = 3^2 + 4^2 = 9 + 16 = 25$.
- Take the square root: $c = \sqrt{25} = 5$.

5. Answer: Foci: (5, 0) and (-5, 0)

- Find c using $c^2 = a^2 + b^2 = 9 + 16 = 25$, so c = 5.
- The hyperbola opens horizontally, so foci are at (±c, 0).
- Foci: (5, 0) and (-5, 0).

6. Answer: Corner points: (3, 4), (3, -4), (-3, 4), (-3, -4)

- From $x^2 = 9$, we get x = ±3 (units along x-axis from center).
- From $y^2 = 16$, we get y = ±4 (units along y-axis from center).
- Draw vertical dashed lines at x = 3 and x = -3, and horizontal dashed lines at y = 4 and y = -4.
- The four corners of the rectangle are (3, 4), (3, -4), (-3, 4), and (-3, -4).

7. Answer: y = (4/3)x and y = -(4/3)x

- For a horizontal hyperbola centered at the origin, asymptotes have slope ±(b/a).
- Here a = 3 and b = 4, so slope = ±4/3.
- Asymptote equations: y = (4/3)x and y = -(4/3)x.

8. Answer: Center: (2, -1), a = 2, b = 3; Vertices: (4, -1) and (0, -1)

- The center is (h, k) = (2, -1).
- $a^2 = 4$, so a = 2; $b^2 = 9$, so b = 3.

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- Since x^2 term is positive, hyperbola opens horizontally.
- Vertices are at (h ± a, k) = (2 ± 2, -1) = (4, -1) and (0, -1).

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9. Answer: Foci: (2 + sqrt(13), -1) and (2 - sqrt(13), -1)

- Find c: $c^2 = a^2 + b^2 = 4 + 9 = 13$, so $c = \sqrt{13}$.
- The hyperbola opens horizontally, so foci are at (h ± c, k).
- Foci: $(2 + \sqrt{13}, -1)$ and $(2 - \sqrt{13}, -1)$.

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10. Answer: $x^2 / 36 - y^2 / 64 = 1$

- Since the hyperbola opens horizontally, use the form $x^2/a^2 - y^2/b^2 = 1$.
- Vertices are at (±6, 0), so a = 6 and $a^2 = 36$.
- Foci are at (±10, 0), so c = 10.
- Use $c^2 = a^2 + b^2$: $100 = 36 + b^2$, so $b^2 = 64$.
- Standard form: $x^2/36 - y^2/64 = 1$.

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