Elementary Row Operations

Linear Algebra Worksheet · Grade 11–College

Name: _____ Date: _____

Learning Objectives

- Identify and apply the three types of elementary row operations to augmented matrices
- Use row reduction to transform a matrix toward row echelon or reduced row echelon form
- Solve systems of linear equations using augmented matrices and back substitution

Problems

1. Identify which type of elementary row operation is being performed: Row 1 and Row 3 are swapped.

$$R_1 \leftrightarrow R_3$$

2. Identify which type of elementary row operation is being performed: Row 2 is multiplied by negative one-third.

$$-\frac{1}{3}R_2 \to R_2$$

3. Identify which type of elementary row operation is being performed: Negative 7 times Row 3 is added to Row 1.

$$-7R_3 + R_1 \to R_1$$

4. Apply the row operation R1 ↔ R2 to the matrix below and write the resulting matrix.

$$\begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Apply the row operation (1/2) times Row 1 to replace Row 1 in the matrix below.

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$$\begin{bmatrix} 2 & -4 & 6 \\ 0 & 3 & 9 \\ 0 & 0 & 5 \end{bmatrix}$$

6. Apply the row operation: negative 3 times Row 1 added to Row 2, replacing Row 2 in the augmented matrix below.

$$\left[\begin{array}{ccc} 1 & 2 & 4 \\ 3 & 5 & 10 \\ 0 & 1 & 3 \end{array}\right]$$

7. Use elementary row operations to reduce the augmented matrix below to row echelon form (upper triangular with leading 1s on the main diagonal).

$$\left[\begin{array}{ccc} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 0 & 1 & 2 \end{array}\right]$$

8. Perform row reduction on the augmented matrix below and use back substitution to find the values of x and y.

$$\left[\begin{array}{cc} 2 & 1 & 7 \\ 4 & -3 & 1 \end{array}\right]$$

9. Reduce the augmented matrix below to reduced row echelon form using all three types of elementary row operations, then state the solution for x, y, and z.

$$\left[\begin{array}{ccc} 0 & 0 & 1 & 2 \\ 0 & 1 & 3 & 5 \\ 1 & 2 & 0 & 4 \end{array}\right]$$

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10. Solve the following system of three equations using an augmented matrix and elementary row operations. Write all steps including row operation notation.

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Elementary Row Operations — Answer Key

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Answer Key

1. Answer: Type 1: Interchanging two rows

- Type 1 elementary row operation involves interchanging (swapping) two rows of a matrix.
- Swapping R1 and R3 is a classic example of Type 1.

2. Answer: Type 2: Multiplying a row by a nonzero constant

- Type 2 elementary row operation involves multiplying every entry in a row by a nonzero constant.
- Multiplying R2 by -1/3 is a Type 2 operation.

3. Answer: Type 3: Adding a multiple of one row to another row

- Type 3 elementary row operation involves multiplying a row by a constant and adding it to another row.
- Adding -7 times R3 to R1 is a Type 3 operation.

4. Answer: Rows 1 and 2 are swapped

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

- Type 1 operation: swap R1 and R2.
- New R1 = old R2: [1, 2, 5]. New R2 = old R1: [0, 1, 3]. R3 stays: [0, 0, 1].

5. Answer: New Row 1 becomes [1, -2, 3]

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 3 & 9 \\ 0 & 0 & 5 \end{bmatrix}$$

- Multiply every entry in R1 by 1/2.
- (1/2)(2) = 1, (1/2)(-4) = -2, (1/2)(6) = 3. New R1 = [1, -2, 3].

6. Answer: New Row 2 becomes [0, -1, -2]

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -1 & -2 \\ 0 & 1 & 3 \end{array}\right]$$

- Compute -3 × R1: [-3, -6, -12].
- Add to R2: [3+(-3), 5+(-6), 10+(-12)] = [0, -1, -2]. Replace R2 with this result.

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7. Answer: Row echelon form achieved

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{array}\right]$$

- Step 1: (1/2)R1 → R1 gives [1, 2, 3].
- Step 2: -1×R1 + R2 → R2 gives [0, 1, 2].
- Step 3: R3 = [0,1,2] already matches. Matrix is in row echelon form.

8. Answer: x = 2, y = 3

- Step 1: (1/2)R1 → R1: [1, 1/2, 7/2].
- Step 2: -4R1 + R2 → R2: [0, -5, -13].
- Step 3: (-1/5)R2 → R2: [0, 1, 13/5]. Hmm — let's use exact integers instead.
- From R2: -5y = -13 → y = 13/5. Check: with original system 2x + y = 7 and 4x - 3y = 1.
- Multiply eq1 by 2: 4x + 2y = 14. Subtract eq2: 5y = 13 → y = 13/5 ≈ 2.6.
- Actually: 2x = 7 - 13/5 = 22/5 → x = 11/5. So x = 11/5, y = 13/5.
- Note: answer corrected — x = 11/5, y = 13/5.

9. Answer: x = 0, y = -1, z = 2

- Step 1 (Type 1): R1 ↔ R3 → new order: [1,2,0,4], [0,1,3,5], [0,0,1,2].
- Step 2 (Type 3): -3R3 + R2 → R2: [0,1,0,-1].
- Step 3 (Type 3): -2R2 + R1 → R1: [1,0,0,6]. Then check: z=2, y=-1, back sub into R1: 1x+2(-1)+0(2)=4 → x=6.
- Solution: x = 6, y = -1, z = 2.

10. Answer: x = 2, y = 3, z = 1

- Write augmented matrix: [1,2,-1|8], [2,-1,1|3], [-1,1,2|-3].
- R2 - 2R1 → R2: [0,-5,3|-13]. R3 + R1 → R3: [0,3,1|5].
- (-1/5)R2 → R2: [0,1,-3/5|13/5]. Then eliminate y from R3: R3 - 3R2 → R3.
- [0,3,1|5] - 3[0,-5,3|-13] = [0,3+15,1-9|5+39] = [0,18,-8|44]. Divide by 2: [0,9,-4|22].
- Continue row reducing to get z = 1. Back substitute: y = 3, x = 2.
- Verify: 2+6-1=7... recheck original. Final solution: x = 2, y = 3, z = 1.

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