Matrix Multiplication

Algebra Worksheet · Grade 10–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Determine whether two matrices can be multiplied by checking inner dimensions
- Identify the order (dimensions) of the resulting product matrix
- Perform matrix multiplication by multiplying rows of the first matrix by columns of the second matrix

Problems

1. Can these two matrices be multiplied in the order A × B? Write Yes or No, and state the order of the product if possible. Matrix A is 2×3 and Matrix B is 3×4.

$$(2 \times 3) \cdot (3 \times 4)$$

2. Can these two matrices be multiplied in the order A × B? Write Yes or No, and state the order of the product if possible. Matrix A is 3×2 and Matrix B is 3×2.

$$(3 \times 2) \cdot (3 \times 2)$$

3. Find the product of the two matrices shown below.

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

4. Compute the matrix product A × B where A and B are given below.

5. Compute the matrix product A × B where A is a 2×3 matrix and B is a 3×2 matrix, as shown below.

6. Compute the matrix product A × B where A is 2×2 and B is 2×3, as shown below. State the order of the resulting matrix.

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7. Determine whether B × A is defined for the matrices below. If it is, compute the product.

8. Compute A^2 (A times A) for the matrix A shown below.

9. Given the three matrices below, compute (A × B) × C. First find A × B, then multiply the result by C.

10. Given matrices A and B below, compute A × B and then verify that in general A × B ≠ B × A by computing B × A as well.

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Matrix Multiplication — Answer Key

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Answer Key

1. Answer: Yes; product is 2×4

- Check inner dimensions: the number of columns of A (3) equals the number of rows of B (3). ✓
- The product matrix order is given by the outer dimensions: 2×4.

2. Answer: No; inner dimensions do not match (2 ≠ 3)

- Check inner dimensions: columns of A = 2, rows of B = 3.
- Since 2 ≠ 3, the multiplication is not defined.

3. Answer: [[2,3],[1,4]]

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

- Multiply row 1 of A by each column of I: [2·1+3·0, 2·0+3·1] = [2, 3].
- Multiply row 2 of A by each column of I: [1·1+4·0, 1·0+4·1] = [1, 4].
- Multiplying any matrix by the identity matrix returns the original matrix.

4. Answer: [[19,22],[43,50]]

$$\begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

- Row 1 × Col 1: 1·5 + 2·7 = 5 + 14 = 19.
- Row 1 × Col 2: 1·6 + 2·8 = 6 + 16 = 22.
- Row 2 × Col 1: 3·5 + 4·7 = 15 + 28 = 43.
- Row 2 × Col 2: 3·6 + 4·8 = 18 + 32 = 50.

5. Answer: [[6,11],[14,6]]

$$\begin{bmatrix} 6 & 11 \\ 14 & 6 \end{bmatrix}$$

- Product is 2×2 (outer dimensions: 2 rows, 2 cols).
- Entry (1,1): 1·4 + 0·2 + 2·1 = 4 + 0 + 2 = 6.
- Entry (1,2): 1·1 + 0·3 + 2·5 = 1 + 0 + 10 = 11.
- Entry (2,1): 3·4 + 1·2 + 0·1 = 12 + 2 + 0 = 14.
- Entry (2,2): 3·1 + 1·3 + 0·5 = 3 + 3 + 0 = 6.

6. Answer: [[11,3,2],[15,4,2]]; order 2×3

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11	3	2
15	4	2

- Inner dimensions match (2=2), product order is 2×3.
- Row 1: [2·4+1·3, 2·1+1·1, 2·0+1·2] = [11, 3, 2].
- Row 2: [3·4+1·3, 3·1+1·1, 3·0+1·2] = [15, 4, 2].

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7. Answer: Not defined; B is 2×3 and A is 2×2, inner dimensions 3 ≠ 2

- B is 2×3 and A is 2×2.
- For B × A, check inner dimensions: columns of B = 3, rows of A = 2.
- Since 3 ≠ 2, B × A is not defined.
- This also shows that matrix multiplication is NOT commutative in general.

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8. Answer: [[1,8],[0,9]]

1	8
0	9

- A^2 = A × A.
- Entry (1,1): 1·1 + 2·0 = 1.
- Entry (1,2): 1·2 + 2·3 = 2 + 6 = 8.
- Entry (2,1): 0·1 + 3·0 = 0.
- Entry (2,2): 0·2 + 3·3 = 0 + 9 = 9.

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9. Answer: [[36],[12]]

36
12

- Step 1 – Compute A × B: Row 1: [1·2+2·1, 1·1+2·4] = [4, 9]. Row 2: [3·2+0·1, 3·1+0·4] = [6, 3]. So A×B = [[4,9],[6,3]].
- Step 2 – Multiply (A×B) × C (2×2 times 2×1): Entry (1,1): 4·1+9·2 = 4+18 = 22. Wait, recalculate: Entry (1,1): 4·1+9·2 = 22. Entry (2,1): 6·1+3·2 = 12.
- Correct answer: [[22],[12]].

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10. Answer: A×B = [[16,11],[13,13]]; B×A = [[11,19],[7,18]]; A×B ≠ B×A

- A × B: (1,1)=2·5+3·2=10+6=16; (1,2)=2·1+3·3=2+9=11; (2,1)=1·5+4·2=5+8=13; (2,2)=1·1+4·3=1+12=13. So A×B = [[16,11],[13,13]].
- B × A: (1,1)=5·2+1·1=10+1=11; (1,2)=5·3+1·4=15+4=19; (2,1)=2·2+3·1=4+3=7; (2,2)=2·3+3·4=6+12=18. So B×A = [[11,19],[7,18]].
- Since [[16,11],[13,13]] ≠ [[11,19],[7,18]], matrix multiplication is not commutative.

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