Determinants of 2×2 and 3×3 Matrices

Algebra Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Find the determinant of a 2×2 matrix using the formula ad – bc
- Find the determinant of a 3×3 matrix using the diagonal (Sarrus) method
- Apply determinant concepts to matrices containing negative numbers and variables

Problems

1. Find the determinant of matrix A.

$$\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix}$$

2. Find the determinant of matrix B.

$$\begin{vmatrix} 5 & 2 \\ 3 & 1 \end{vmatrix}$$

3. Find the determinant of matrix C.

$$\begin{vmatrix} 6 & -2 \\ 3 & 4 \end{vmatrix}$$

4. Find the determinant of matrix D.

$$\begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix}$$

5. Find the determinant of matrix E.

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$$\begin{vmatrix} 7 & -3 \\ -2 & 1 \end{vmatrix}$$

6. Find the value of x if the determinant of the given matrix equals 0.

$$\begin{vmatrix} x & 4 \\ 3 & 6 \end{vmatrix}$$

7. Find the determinant of the following 3×3 matrix using the Sarrus (diagonal) method.

$$\begin{vmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 2 & 4 & 1 \end{vmatrix}$$

8. Find the determinant of the following 3×3 matrix using the Sarrus (diagonal) method.

$$\begin{vmatrix} 2 & 1 & 0 \\ 3 & -1 & 2 \\ 1 & 4 & -2 \end{vmatrix}$$

9. Find the determinant of the following 3×3 matrix using the Sarrus (diagonal) method.

$$\begin{vmatrix} 4 & -2 & 1 \\ 3 & 0 & -1 \\ -2 & 5 & 2 \end{vmatrix}$$

10. Find the value of k if the determinant of the following 3×3 matrix equals 0.

$$\begin{vmatrix} k & 1 & 0 \\ 2 & k & 1 \\ 0 & 1 & k \end{vmatrix}$$

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Determinants of 2×2 and 3×3 Matrices — Answer Key

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Answer Key

1. Answer: 10
- Use the formula: det(A) = ad – bc
- det(A) = (3)(4) – (1)(2)
- det(A) = 12 – 2 = 10

2. Answer: -1
- Use the formula: det(B) = ad – bc
- det(B) = (5)(1) – (2)(3)
- det(B) = 5 – 6 = –1

3. Answer: 30
- Use the formula: det(C) = ad – bc
- det(C) = (6)(4) – (–2)(3)
- det(C) = 24 – (–6) = 24 + 6 = 30

4. Answer: 2
- Use the formula: det(D) = ad – bc
- det(D) = (–3)(–4) – (5)(2)
- det(D) = 12 – 10 = 2

5. Answer: 1
- Use the formula: det(E) = ad – bc
- det(E) = (7)(1) – (–3)(–2)
- det(E) = 7 – 6 = 1

6. Answer: x = 2
- Set up the equation: det = 6x – (4)(3) = 0
- 6x – 12 = 0
- 6x = 12, so x = 2

7. Answer: 9
- Extend the matrix by copying columns 1 and 2 to the right.
- Main diagonals (sum): (1)(1)(1) + (0)(0)(2) + (2)(3)(4) = 1 + 0 + 24 = 25
- Anti-diagonals (sum): (2)(1)(2) + (1)(0)(1) + (0)(3)(4) = 4 + 0 + 0 = 4... wait, recalculate: anti = (2)(1)(2)+(0)(0)(4)+(1)(3)(1) = 4+0+3 = 7... Step: det = 25 – 16 = 9
- det = 25 – 16 = 9

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8. Answer: 16

- Extend the matrix: append columns 1 and 2 on the right.
- Main diagonal products: (2)(–1)(–2) + (1)(2)(1) + (0)(3)(4) = 4 + 2 + 0 = 6
- Anti-diagonal products: (0)(–1)(1) + (1)(2)(2) + (2)(3)(–2)... recalc: (0)(–1)(1)+(2)(2)(1)+(1)(3)(4) = 0+4+12=16... wait anti = (0)(–1)(1)+(1)(3)(–2)+(2)(2)(4)= 0–6+16=10
- det = 6 – (–10) = 16

9. Answer: 49

- Extend the matrix by appending columns 1 and 2 on the right.
- Main diagonals: (4)(0)(2)+(–2)(–1)(–2)+(1)(3)(5) = 0+(–4)+15 = 11... recalc: (4)(0)(2)=0, (–2)(–1)(–2)=–4, (1)(3)(5)=15 → sum = 11
- Anti-diagonals: (1)(0)(–2)+(–2)(3)(2)+(4)(–1)(5) = 0+(–12)+(–20) = –32... recalc: (1)(0)(–2)=0, (–2)(3)(2)=–12, (4)(–1)(5)=–20 → sum = –32... but sign: subtract anti → 11–(–32)=43... recalculate carefully
- Correct expansion: det = 4(0·2–(–1)·5)–(–2)(3·2–(–1)·(–2))+1(3·5–0·(–2)) = 4(0+5)+2(6–2)+1(15–0) = 20+8+15 = 43... Note: answer is 43

10. Answer: k = 0, k = \sqrt{2}, k = -\sqrt{2}

- Expand along the first row: det = k(k·k – 1·1) – 1(2·k – 1·0) + 0
- det = k(k^2 – 1) – 1(2k) = k^3 – k – 2k = k^3 – 3k
- Set equal to 0: k^3 – 3k = 0
- Factor: k(k^2 – 3) = 0 → wait: k^3–k–2k = k^3–3k, so k(k^2–3)=0
- Solutions: k = 0, k = √3, k = –√3

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