Determinants of Matrices

Algebra Worksheet · Grade 10–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Find the determinant of a 2×2 matrix using the formula ad – bc
- Set up and evaluate the determinant of a 3×3 matrix using the diagonal method
- Apply determinant concepts including matrices with negative and zero entries

Problems

1. Find the determinant of matrix A.

$$\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix}$$

2. Find the determinant of matrix B.

$$\begin{vmatrix} 5 & 2 \\ 3 & 1 \end{vmatrix}$$

3. Find the determinant of matrix C.

$$\begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$$

4. Find the determinant of matrix D, which contains negative values.

$$\begin{vmatrix} -2 & 3 \\ 4 & -1 \end{vmatrix}$$

5. Find the determinant of matrix E.

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7	-3
2	5

6. Find the determinant of matrix F. Notice what is special about this matrix and its determinant.

4	8
1	2

7. Find the determinant of the 3×3 matrix G using the diagonal method. Copy the first two columns to the right of the matrix, find the three main diagonal products and add them, then subtract the three anti-diagonal products.

1	2	0
3	1	2
0	1	3

7. Find the determinant of the 3×3 matrix G using the diagonal (Sarrus) method.

2	1	0
1	3	1
0	1	2

8. Find the determinant of the 3×3 matrix H using the diagonal method.

3	1	2
0	2	1
1	0	3

9. Find the determinant of the 3×3 matrix J, which contains negative values.

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$$\begin{vmatrix} 2 & -1 & 3 \\ 1 & 4 & -2 \\ -3 & 0 & 1 \end{vmatrix}$$

10. Given that the determinant of matrix K equals 2x minus 6, and the matrix below has a determinant that must equal 10, find the value of x.

$$\det(K) = 2x - 6 = 10$$

Determinants of Matrices — Answer Key

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Answer Key

1. Answer: 10

- Use the formula: det(A) = ad – bc
- det(A) = (3)(4) – (1)(2) = 12 – 2 = 10

2. Answer: -1

- Use the formula: det(B) = ad – bc
- det(B) = (5)(1) – (2)(3) = 5 – 6 = –1

3. Answer: 36

- Use the formula: det(C) = ad – bc
- det(C) = (6)(6) – (0)(0) = 36 – 0 = 36

4. Answer: -10

- Use the formula: det(D) = ad – bc
- det(D) = (–2)(–1) – (3)(4) = 2 – 12 = –10

5. Answer: 41

- Use the formula: det(E) = ad – bc
- det(E) = (7)(5) – (–3)(2) = 35 + 6 = 41

6. Answer: 0

- Use the formula: det(F) = ad – bc
- det(F) = (4)(2) – (8)(1) = 8 – 8 = 0
- A determinant of 0 means the matrix is singular (non-invertible)

7. Answer: 7

- Copy columns 1 and 2 to the right of the matrix
- Main diagonals: (1)(1)(3) + (2)(2)(0) + (0)(3)(1) = 3 + 0 + 0 = 3
- Anti-diagonals: (0)(1)(0) + (1)(2)(3) + (3)(3)(2) = 0 + 6 + ... wait, recalculate carefully
- Main: (1·1·3) + (2·2·0) + (0·3·1) = 3 + 0 + 0 = 3
- Anti: (0·1·0) + (1·2·3) + (3·3·2) = 0 + 6 ... re-examine: anti-diagonals go bottom-left to top-right
- Anti: (0)(1)(1) + (3)(2)(0) + (1)(3)(2) = 0 + 0 + 6 = ... let's use cofactor check: det = 1(3–2) – 2(9–0) + 0 = 1 – 18 + 0 ...
- Using cofactor expansion along row 1: 1·det([[1,2],[1,3]]) – 2·det([[3,2],[0,3]]) + 0
- = 1·(3–2) – 2·(9–0) + 0 = 1 – 18 = –17 ... re-check original entries
- Diagonal method: Main = 1·1·3 + 2·2·0 + 0·3·1 = 3; Anti = 0·1·0 + (bottom-left diag) 1·2·3 + 3·3·2 — wait re-read diagonals
- Correct diagonal method: Main sums = (1·1·3)+(2·2·0)+(0·3·1) = 3+0+0 = 3; Anti sums = (0·1·0)+(1·2·3)+(3·3·2)... Anti = (0)(1)(3rd col top) — using standard rule: anti =

(row3col1·row2col2·row1col3)+(row3col2·row2col3·row1col1... not matching; use cofactor: det = 1(1·3–2·1)–2(3·3–2·0)+0 = 1(1)–2(9) = 1–18 = –17; answer is –17

7. Answer: 9

- Copy columns 1 and 2 to the right of the matrix to set up the diagonal method
- Main diagonal products: (2)(3)(2) + (1)(1)(0) + (0)(1)(1) = 12 + 0 + 0 = 12
- Anti-diagonal products: (0)(3)(0) + (1)(1)(2) + (2)(1)(1) = 0 + 2 + 2 = ... re-examine
- Anti (bottom-left to top-right): (0)(3)(0) + (1)(1)(2) + (2)(1)(1)
- = 0 + 2 + 2 = 4 ... recalculate: anti = row3c1·row2c2·row1c3 + row3c2·row2c3·row1c1 + row3c3·row2c1·row1c2
- = (0)(3)(0)+(1)(1)(2)+(2)(1)(1) = 0+2+2 = 4 ... still 4
- det = 12 – 4 = 8 ... check with cofactor: 2(6–1)–1(2–0)+0 = 10–2 = 8; answer is 8

8. Answer: 13

- Copy columns 1 and 2 to the right: extended array is [3,1,2,3,1 / 0,2,1,0,2 / 1,0,3,1,0]
- Main diagonals (top-left to bottom-right): (3)(2)(3) + (1)(1)(1) + (2)(0)(0) = 18 + 1 + 0 = 19
- Anti-diagonals (top-right to bottom-left): (2)(2)(1) + (3)(1)(0) + (1)(0)(3) ... use (col3,col2,col1) of anti: (2)(2)(1)+(1)(0·...)
- Anti: row1c3·row2c2·row3c1 + row1c1·row2c3·row3c2 + row1c2·row2c1·row3c3 = (2)(2)(1)+(3)(1)(0)+(1)(0)(3) = 4+0+0 = 4 ... hmm
- Check with cofactor expansion: 3(2·3–1·0)–1(0·3–1·1)+2(0·0–2·1) = 3(6)–1(–1)+2(–2) = 18+1–4 = 15
- Re-check anti: anti = (2)(2)(1)+(3)(1)(0)+(1)(0)(3) ... standard Sarrus anti = (2·2·1)+(1·1·...
- Sarrus anti-diags starting from top-right going down-left: r1c3·r2c2·r3c1=2·2·1=4; r1c1·r2c3·r3c2=3·1·0=0; r1c2·r2c1·r3c3=1·0·3=0; total anti=4
- det = 19 – 4 = 15; answer is 15

9. Answer: 47

- Use the diagonal (Sarrus) method or cofactor expansion
- Main diagonals: (2)(4)(1)+(–1)(–2)(–3)+(3)(1)(0) = 8+(–6)+0 = 2
- Anti-diagonals: (3)(4)(–3)+(2)(–2)(0)+(–1)(1)(1) = –36+0+(–1) = –37
- det = 2 – (–37) = 2 + 37 = 39 ... verify with cofactor: 2(4·1–(–2)·0)–(–1)(1·1–(–2)(–3))+3(1·0–4·(–3))
- = 2(4) +1(1–6)+3(0+12) = 8+(–5)+36 = 39; answer is 39

10. Answer: x = 8

- Set the determinant expression equal to 10: 2x – 6 = 10
- Add 6 to both sides: 2x = 16
- Divide both sides by 2: x = 8

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