# Inverse of 2×2 Matrices

Linear Algebra Worksheet · Grade 10–12

Name: _____ Date: _____

**Learning Objectives**

- Verify that two matrices are inverses by showing AB = BA = I
- Apply the inverse formula to find the inverse of a 2×2 matrix
- Calculate the determinant of a 2×2 matrix and use it in the inverse formula

## Problems

**1.** Write the 2×2 identity matrix.

**2.** Find the determinant of the matrix below.

$$\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix}$$

**3.** Find the determinant of the matrix below.

$$\begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix}$$

**4.** Using the inverse formula, transform (but do not scale) the matrix below by swapping the main diagonal entries and changing the signs of the off-diagonal entries.

$$\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$$

**5.** Find the inverse of the matrix below.

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$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

**6.** Find the inverse of the matrix below.

$$\begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$$

**7.** Find the inverse of the matrix below.

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix}$$

**8.** Verify that B is the inverse of A by computing both A times B and B times A, and confirming both equal the identity matrix. A is the first matrix and B is the second matrix shown below.

**9.** Determine whether the matrix below has an inverse. If it does, find it. If not, explain why.

$$\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$$

**10.** Matrix A and matrix B are given below. First find the inverse of A. Then verify your answer by showing that A times A-inverse equals the identity matrix.

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# Inverse of 2×2 Matrices — Answer Key

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## Answer Key

**1. Answer: [[1,0],[0,1]]**

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- The identity matrix I has 1s on the main diagonal and 0s elsewhere.
- For a 2×2 matrix: first row is [1, 0] and second row is [0, 1].

**2. Answer: 10**

- det(A) = ad - bc = (4)(3) - (2)(1)
- = 12 - 2 = 10

**3. Answer: 4**

- det(A) = ad - bc = (3)(2) - (-2)(-1)
- = 6 - 2 = 4

**4. Answer: [[1,-3],[-2,5]]**

$$\begin{bmatrix} 1 & -3 \\ -2 & 5 \end{bmatrix}$$

- Swap the main diagonal: a=5 and d=1 become d=1 and a=5 in swapped positions.
- Change signs of off-diagonal: b=3 becomes -3, c=2 becomes -2.
- Transformed matrix: [[1, -3], [-2, 5]]

**5. Answer: [[1,-1],[-1,2]]**

$$\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

- det(A) = (2)(1) - (1)(1) = 2 - 1 = 1
- A■¹ = (1/1) * [[1, -1], [-1, 2]]
- A■¹ = [[1, -1], [-1, 2]]

**6. Answer: [[2,-7],[-1,4]]**

$$\begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$$

- det(A) = (4)(2) - (7)(1) = 8 - 7 = 1

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- A■¹ = (1/1) * [[2, -7], [-1, 4]]
- A■¹ = [[2, -7], [-1, 4]]

### 7. Answer: (1/4)[[2,2],[1,3]]

$$\begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}$$

- det(A) = (3)(2) - (-2)(-1) = 6 - 2 = 4
- Transformed matrix: [[2, 2], [1, 3]]
- A■¹ = (1/4) * [[2, 2], [1, 3]] = [[1/2, 1/2], [1/4, 3/4]]

### 8. Answer: AB = BA = I, so B is the inverse of A

- AB: row1×col1 = (2)(3)+(1)(-5)=1, row1×col2 = (2)(-1)+(1)(2)=0
- row2×col1 = (5)(3)+(3)(-5)=0, row2×col2 = (5)(-1)+(3)(2)=1 → AB = I
- BA: row1×col1 = (3)(2)+(-1)(5)=1, row1×col2 = (3)(1)+(-1)(3)=0
- row2×col1 = (-5)(2)+(2)(5)=0, row2×col2 = (-5)(1)+(2)(3)=1 → BA = I
- Since AB = BA = I, B is the inverse of A.

### 9. Answer: No inverse exists; the determinant is 0 (singular matrix).

- det(A) = (2)(2) - (4)(1) = 4 - 4 = 0
- Since the determinant equals 0, the matrix is singular.
- A singular matrix does not have an inverse.

### 10. Answer: A■¹ = [[2,-3],[-3,5]]

$$\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

- det(A) = (5)(2) - (3)(3) = 10 - 9 = 1
- A■¹ = (1/1) * [[2, -3], [-3, 5]] = [[2, -3], [-3, 5]]
- Verify: A·A■¹ row1×col1 = (5)(2)+(3)(-3)=10-9=1 ✓
- row1×col2 = (5)(-3)+(3)(5)=-15+15=0 ✓
- row2×col1 = (3)(2)+(2)(-3)=6-6=0 ✓
- row2×col2 = (3)(-3)+(2)(5)=-9+10=1 ✓ → A·A■¹ = I

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