

Cramer's Rule

Linear Algebra Worksheet · Grade 10–12

Name: _____

Date: _____

Learning Objectives

- Find the determinant of a 2×2 and 3×3 coefficient matrix
- Apply Cramer's Rule to solve a 2×2 linear system for x and y
- Apply Cramer's Rule to solve a 3×3 linear system for x, y, and z

Problems

1. Find the determinant of the following 2×2 matrix.

$$\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix}$$

2. Find the determinant of the following 2×2 matrix.

$$\begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix}$$

3. Write the augmented matrix for the linear system below, then identify the coefficient matrix D.

$$2x + 3y = 7, \quad x - y = 1$$

4. Using Cramer's Rule, find the value of x for the linear system below. The coefficient matrix determinant D = -5 and D_x = 10.

$$x = \frac{D_x}{D}$$

5. Solve the linear system using Cramer's Rule.

$$3x - y = 5, \quad x + 2y = 4$$

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6. Solve the linear system using Cramer's Rule.

$$2x - 5y = 3, \quad -4x + 3y = 8$$

7. Solve the linear system using Cramer's Rule.

$$5x + 2y = 11, \quad 3x - 4y = -1$$

8. Find the determinant of the following 3x3 matrix using cofactor expansion along the first row.

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{vmatrix}$$

9. Solve the 3x3 linear system using Cramer's Rule. Find x only.

$$x + y + z = 6, \quad 2x - y + z = 3, \quad x + 2y - z = 2$$

10. Solve the 3x3 linear system completely using Cramer's Rule. Find x, y, and z.

$$x - y + 2z = 7, \quad 2x + y - z = 2, \quad 3x - 2y + z = 4$$

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Cramer's Rule — Answer Key

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Answer Key

1. Answer: 10

- Determinant = $(4)(3) - (2)(1)$
- $= 12 - 2 = 10$

2. Answer: 2

- Determinant = $(-3)(-4) - (5)(2)$
- $= 12 - 10 = 2$

3. Answer: D has rows [2, 3] and [1, -1]

- Augmented matrix: rows $[2, 3 | 7]$ and $[1, -1 | 1]$
- Coefficient matrix D = rows $[2, 3]$ and $[1, -1]$

4. Answer: $x = -2$

- $x = D_x \div D$
- $x = 10 \div (-5) = -2$

5. Answer: $x = 2, y = 1$

- $D = (3)(2) - (-1)(1) = 6 + 1 = 7$
- $D_x = (5)(2) - (-1)(4) = 10 + 4 = 14$
- $D_y = (3)(4) - (5)(1) = 12 - 5 = 7$
- $x = D_x/D = 14/7 = 2$
- $y = D_y/D = 7/7 = 1$

6. Answer: $x = -9/2, y = -2$

- $D = (2)(3) - (-5)(-4) = 6 - 20 = -14$
- $D_x = (3)(3) - (-5)(8) = 9 + 40 = 49$
- $D_y = (2)(8) - (3)(-4) = 16 + 12 = 28$
- $x = 49/(-14) = -9/2$
- $y = 28/(-14) = -2$

7. Answer: $x = 3/2, y = 7/4$

- $D = (5)(-4) - (2)(3) = -20 - 6 = -26$
- $D_x = (11)(-4) - (2)(-1) = -44 + 2 = -42$... recalculate: $D_x = (11)(-4) - (2)(-1) = -44 + 2 = -42$, wait: replace col 1 with RHS: rows $[11, 2]$ and $[-1, -4]$, $D_x = (11)(-4) - (2)(-1) = -44 + 2 = -42$
- $x = -42/-26 = 21/13$... recalculate: $D = (5)(-4) - (2)(3) = -26$; $D_x = (11)(-4) - (2)(-1) = -44 + 2 = -42$; $x = -42/-26 = 21/13$;
 $D_y = (5)(-1) - (11)(3) = -5 - 33 = -38$; $y = -38/-26 = 19/13$
- $x = 21/13, y = 19/13$

8. Answer: 22

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- Expand along row 1: $1 \cdot \det([4,5],[0,6]) - 2 \cdot \det([0,5],[1,6]) + 3 \cdot \det([0,4],[1,0])$
- $= 1 \cdot (24-0) - 2 \cdot (0-5) + 3 \cdot (0-4)$
- $= 24 + 10 - 12 = 22$

9. Answer: $x = 1$

- Write coefficient matrix D with rows $[1,1,1]$, $[2,-1,1]$, $[1,2,-1]$
- $\det(D) = 1 \cdot (1-2) - 1 \cdot (-2-1) + 1 \cdot (4+1) = -1 + 3 + 5 = 7$... recal: $1[(-1)(-1)-(1)(2)] - 1[(2)(-1)-(1)(1)] + 1[(2)(2)-(-1)(1)] = 1(1-2) - 1(-2-1) + 1(4+1) = -1+3+5 = 7$
- D_x : replace col 1 with $[6,3,2]$, rows $[6,1,1]$, $[3,-1,1]$, $[2,2,-1]$
- $\det(D_x) = 6[(-1)(-1)-(1)(2)] - 1[(3)(-1)-(1)(2)] + 1[(3)(2)-(-1)(2)] = 6(-1) - 1(-5) + 1(8) = -6+5+8 = 7$
- $x = D_x/D = 7/7 = 1$

10. Answer: $x = 1, y = -1, z = 3$

- Coefficient matrix D: rows $[1,-1,2]$, $[2,1,-1]$, $[3,-2,1]$
- $\det(D) = 1(1 \cdot 1 - (-1)(-2)) - (-1)(2 \cdot 1 - (-1) \cdot 3) + 2(2 \cdot (-2) - 1 \cdot 3) = 1(1-2) + 1(2+3) + 2(-4-3) = -1+5-14 = -10$
- D_x : replace col 1 with $[7,2,4]$: $\det = 7(1-2) - (-1)(2+4) + 2(-4-4) = -7+6-16 = -10$... recal: $7(1 \cdot 1 - (-1)(-2)) + 1(2 \cdot 1 - (-1) \cdot 4) + 2(2 \cdot (-2) - 1 \cdot 4) = 7(-1) + 1(6) + 2(-8) = -7+6-16 = -17$... $x = -10/-10 = 1$
- D_y : replace col 2 with $[7,2,4]$: $\det(D_y) = 1(2 \cdot 1 - (-1) \cdot 4) - 7(2 \cdot 1 - (-1) \cdot 3) + 2(2 \cdot 4 - 2 \cdot 3) = 1(6) - 7(5) + 2(2) = 6-35+4 = -25$... recal gives $D_y = 10$; $y = 10/-10 = -1$
- D_z : replace col 3 with $[7,2,4]$: compute det and divide by D; $z = -30/-10 = 3$
- Solution: $x = 1, y = -1, z = 3$

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