

Mathematical Induction

Algebra & Proof Worksheet · Grade 10–12

Name: _____

Date: _____

Learning Objectives

- Prove a statement is true for $n = 1$ (base case)
- Assume the statement is true for $n = k$ (inductive hypothesis)
- Prove the statement is true for $n = k + 1$ (inductive step)

Problems

1. In Step 1 of mathematical induction, what value do you substitute for n to verify the base case?

2. Verify the base case ($n = 1$) for the formula below:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

3. Write the inductive hypothesis (Step 2) for the formula below by replacing n with k :

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

4. Write the expression that must be proved in Step 3 (the inductive step) for the formula below, when n is replaced by $k + 1$:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

5. Complete the inductive step by simplifying the left side below using the inductive hypothesis, then confirm it equals the right side:

$$\frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2}$$

6. Verify the base case ($n = 1$) for the formula below:

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$$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

7. Write the inductive hypothesis and the goal of the inductive step for the formula below:

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

8. Complete the inductive step to prove the formula below. Start from the inductive hypothesis and add the next term:

$$k^2 + (2k + 1) = (k + 1)^2$$

9. Use all three steps of mathematical induction to prove the formula below:

$$1 + 2 + 3 + \cdots + n = \frac{n(n + 1)}{2}$$

10. Use all three steps of mathematical induction to prove the formula below:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n + 1)(2n + 1)}{6}$$

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Mathematical Induction — Answer Key

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Answer Key

1. Answer: $n = 1$

- Step 1: The base case always begins by testing the first natural number.
- Step 2: Substitute $n = 1$ into both sides of the equation to confirm they are equal.

2. Answer: Both sides equal 1, so the base case holds.

- Step 1: Substitute $n = 1$ on the left side: the sum is just 1.
- Step 2: Substitute $n = 1$ on the right side: $\frac{1(1+1)}{2} = \frac{2}{2} = 1$.
- Step 3: Since both sides equal 1, the base case is verified.

3. Answer: $1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$

- Step 1: Assume the formula is true for $n = k$.
- Step 2: Replace every n with k to get: $1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$.

4. Answer: $1 + 2 + \dots + (k+1) = \frac{(k+1)(k+2)}{2}$

- Step 1: Replace every n with $k + 1$.
- Step 2: Left side becomes: $1 + 2 + 3 + \dots + k + (k+1)$.
- Step 3: Right side becomes: $\frac{(k+1)(k+2)}{2}$.

5. Answer: $\frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2}$. Proved.

- Step 1: Write $k+1$ as a fraction over 2: $\frac{2(k+1)}{2}$.
- Step 2: Add: $\frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$.
- Step 3: This matches the right side, so the inductive step is complete.

6. Answer: Both sides equal 1, so the base case holds.

- Step 1: Left side with $n = 1$: the only term is $2(1) - 1 = 1$.
- Step 2: Right side with $n = 1$: $1^2 = 1$.
- Step 3: Both sides equal 1, base case verified.

7. Answer: Hypothesis: sum to $k = k^2$. Goal: prove sum to $k+1 = (k+1)^2$.

- Step 1 (Hypothesis): Assume $1 + 3 + 5 + \dots + (2k-1) = k^2$.
- Step 2 (Goal): Show that $1 + 3 + 5 + \dots + (2k-1) + (2k+1) = (k+1)^2$.

8. Answer: $k^2 + 2k + 1 = (k+1)^2$. Proved.

- Step 1: Using the inductive hypothesis, the left side is $k^2 + (2k+1)$.
- Step 2: Expand: $k^2 + 2k + 1$.
- Step 3: Factor: $(k+1)^2$, which matches the right side.

9. Answer: All three steps verified. The formula is true for all natural numbers n .

- Step 1 (Base case, $n=1$): Left side = 1. Right side = $\frac{1 \cdot 2}{2} = 1$. True.

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- Step 2 (Hypothesis): Assume $1 + 2 + \dots + k = \frac{k(k+1)}{2}$.
- Step 3 (Inductive step): Add $(k+1)$ to both sides: $\frac{k(k+1)}{2} + (k+1) = \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$.
- Step 4: This matches the formula with $n = k+1$, so the proof is complete by mathematical induction.

10. Answer: All three steps verified. The formula holds for all natural numbers n .

- Step 1 (Base case, $n=1$): Left side = 1. Right side = $\frac{1 \cdot 2 \cdot 3}{6} = 1$. True.
- Step 2 (Hypothesis): Assume $1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$.
- Step 3 (Inductive step): Add $(k+1)^2$: $\frac{k(k+1)(2k+1)}{6} + (k+1)^2$.
- Step 4: Factor out $(k+1)$: $\frac{(k+1)[k(2k+1) + 6(k+1)]}{6} = \frac{(k+1)(2k^2+7k+6)}{6}$.
- Step 5: Factor the trinomial: $2k^2+7k+6 = (k+2)(2k+3)$.
- Step 6: Result is $\frac{(k+1)(k+2)(2k+3)}{6}$, which matches the formula with $n = k+1$. Proof complete.

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