Expanding Binomials Using Pascal's Triangle

Algebra Worksheet · Grade 9–11

Name: ________________________________ Date: __________________

Learning Objectives

- Build Pascal's Triangle and identify coefficients for any power
- Expand binomial expressions using Pascal's Triangle coefficients
- Apply the binomial expansion process with descending and ascending exponent patterns

Problems

1. Write the row of Pascal's Triangle that corresponds to the exponent shown below.

$$n = 4$$

2. Write the row of Pascal's Triangle that corresponds to the exponent shown below.

$$n = 5$$

3. How many terms will the expanded form have? State the Pascal's Triangle row you would use.

$$(a + b)^6$$

4. Expand the binomial below using Pascal's Triangle.

$$(x + y)^3$$

5. Expand the binomial below using Pascal's Triangle.

$$(x + 2)^3$$

6. Expand the binomial below using Pascal's Triangle.

$$(x + 3)^4$$

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7. Expand the binomial below using Pascal's Triangle.

$(2x + 1)^4$

8. Expand the binomial below using Pascal's Triangle.

$(x - 2)^4$

9. Expand the binomial below using Pascal's Triangle.

$(3x - 2)^4$

10. Expand the binomial below using Pascal's Triangle.

$(2x + 3)^5$

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Expanding Binomials Using Pascal's Triangle — Answer Key

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Answer Key

1. Answer: 1, 4, 6, 4, 1

- Row 0: 1
- Row 1: 1 1
- Row 2: 1 2 1
- Row 3: 1 3 3 1
- Row 4: 1 4 6 4 1

2. Answer: 1, 5, 10, 10, 5, 1

- Start from Row 4: 1 4 6 4 1
- Add adjacent pairs: 1+4=5, 4+6=10, 6+4=10, 4+1=5
- Place 1s on the ends: 1 5 10 10 5 1

3. Answer: 7 terms; row: 1, 6, 15, 20, 15, 6, 1

- Number of terms = exponent + 1 = 6 + 1 = 7
- Build from Row 5: 1 5 10 10 5 1
- Row 6: 1 6 15 20 15 6 1

4. Answer: $x^3 + 3x^2y + 3xy^2 + y^3$

- Row 3 coefficients: 1, 3, 3, 1
- x exponents descend: 3, 2, 1, 0 — y exponents ascend: 0, 1, 2, 3
- Combine: $1 \cdot x^3 \cdot y^0 + 3 \cdot x^2 \cdot y^1 + 3 \cdot x^1 \cdot y^2 + 1 \cdot x^0 \cdot y^3$
- Simplify: $x^3 + 3x^2y + 3xy^2 + y^3$

5. Answer: $x^3 + 6x^2 + 12x + 8$

- Row 3 coefficients: 1, 3, 3, 1
- Terms: $1 \cdot x^3 \cdot 2^0 + 3 \cdot x^2 \cdot 2^1 + 3 \cdot x^1 \cdot 2^2 + 1 \cdot x^0 \cdot 2^3$
- Simplify powers of 2: $2^0=1$, $2^1=2$, $2^2=4$, $2^3=8$
- Multiply by coefficients: $x^3 + 6x^2 + 12x + 8$

6. Answer: $x^4 + 12x^3 + 54x^2 + 108x + 81$

- Row 4 coefficients: 1, 4, 6, 4, 1
- Terms: $x^4 \cdot 3^0 + 4 \cdot x^3 \cdot 3^1 + 6 \cdot x^2 \cdot 3^2 + 4 \cdot x \cdot 3^3 + 3^4$
- Simplify: $x^4 + 4 \cdot 3x^3 + 6 \cdot 9x^2 + 4 \cdot 27x + 81$
- Result: $x^4 + 12x^3 + 54x^2 + 108x + 81$

7. Answer: $16x^4 + 32x^3 + 24x^2 + 8x + 1$

- Row 4 coefficients: 1, 4, 6, 4, 1

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- Terms: $(2x)^4 + 4(2x)^3(1) + 6(2x)^2(1)^2 + 4(2x)(1)^3 + (1)^4$
- Simplify: $16x^4 + 4\cdot 8x^3 + 6\cdot 4x^2 + 4\cdot 2x + 1$
- Result: $16x^4 + 32x^3 + 24x^2 + 8x + 1$

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8. Answer: $x^4 - 8x^3 + 24x^2 - 32x + 16$

- Rewrite as $(x + (-2))^4$; Row 4 coefficients: 1, 4, 6, 4, 1
- Terms: $x^4 + 4x^3(-2) + 6x^2(-2)^2 + 4x(-2)^3 + (-2)^4$
- Simplify powers: -2, 4, -8, 16
- Result: $x^4 - 8x^3 + 24x^2 - 32x + 16$

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9. Answer: $81x^4 - 216x^3 + 216x^2 - 96x + 16$

- Rewrite as $(3x + (-2))^4$; Row 4 coefficients: 1, 4, 6, 4, 1
- Terms: $(3x)^4 + 4(3x)^3(-2) + 6(3x)^2(-2)^2 + 4(3x)(-2)^3 + (-2)^4$
- Simplify: $81x^4 + 4(-54x^3) + 6(9x^2)(4) + 4(3x)(-8) + 16$
- Result: $81x^4 - 216x^3 + 216x^2 - 96x + 16$

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10. Answer: $32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$

- Row 5 coefficients: 1, 5, 10, 10, 5, 1
- Terms: $(2x)^5 + 5(2x)^4(3) + 10(2x)^3(3)^2 + 10(2x)^2(3)^3 + 5(2x)(3)^4 + (3)^5$
- Simplify each: $32x^5 + 5\cdot 16x^4\cdot 3 + 10\cdot 8x^3\cdot 9 + 10\cdot 4x^2\cdot 27 + 5\cdot 2x\cdot 81 + 243$
- Result: $32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$
