

Statistics: Conditional Probability

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DIRECTIONS

Apply the conditional probability formula $P(A|B) = P(A \cap B)/P(B)$. For table problems, restrict the sample space to the given condition. Check independence: A and B are independent if $P(A|B) = P(A)$.

- 1 From the table: 60 males (30 passed), 40 females (28 passed). Find:

$$P(\text{passed} | \text{male})$$

Answer: _____

- 2 Same table: 30 males passed, 28 females passed, 100 total. Find:

$$P(\text{female} | \text{passed})$$

Answer: _____

- 3 Are A and B independent? Check:

$$P(A) = 0.40, \quad P(A|B) = 0.40$$

Answer: _____

- 4 Find $P(A|B)$:

$$P(A|B) = 0.35$$

Answer: _____

- 5 Find $P(B|A)$:

$$P(A) = 0.50, \quad P(B) = 0.60, \quad P(A \cap B) = 0.30$$

Answer: _____

- 6 Survey: 100 people, 40 under age 30, 28 of them like coffee. Find:

$$P(\text{likes coffee} | \text{under 30})$$

Answer: _____

- 7 Find $P(A \cap B)$:

$$P(B|A) = 0.70, \quad P(A) = 0.30$$

Answer: _____

- 8 Test independence: does $P(A \cap B) = P(A) \cdot P(B)$?

$$P(A) = 0.50, \quad P(B) = 0.40, \quad P(A \cap B) = 0.25$$

Answer: _____

Answer Key & Solutions

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TEACHER NOTES

Formula: $P(A|B) = P(A \cap B)/P(B)$. Independence: $P(A|B) = P(A)$, equivalent to $P(A \cap B) = P(A) \cdot P(B)$.
Table problems: always identify the restricted row/column total as the denominator.

- 1 From the table: 60 males (30 passed), 40 females (28 passed). Find:

$$P(\text{passed} | \text{male})$$

$$= \frac{30}{60} = 0.50$$

Restrict sample space to males (60). Of those, 30 passed: $30/60 = 0.50$.

- 2 Same table: 30 males passed, 28 females passed, 100 total. Find:

$$P(\text{female} | \text{passed})$$

$$= \frac{28}{58} = \frac{14}{29} \approx 0.483$$

Total passed = $30+28 = 58$. Of those, 28 are female: $28/58 \approx 0.483$.

- 3 Are A and B independent? Check:

$$P(A) = 0.40, \quad P(A|B) = 0.40$$

$$= \text{Yes — Independent}$$

Events are independent when $P(A|B) = P(A)$. Knowing B gives no new info about A.

- 4 Find $P(A'|B)$:

$$P(A|B) = 0.35$$

$$= P(A'|B) = 1 - 0.35 = 0.65$$

Within the sample space restricted to B, A and A' must sum to 1.

- 5 Find $P(B|A)$:

$$P(A) = 0.50, \quad P(B) = 0.60, \quad P(A \cap B) = 0.30$$

$$= P(B|A) = \frac{0.30}{0.50} = 0.60$$

$P(B|A) = P(A \cap B)/P(A) = 0.30/0.50 = 0.60$. Here $P(B|A) = P(B)$, so independent.

- 6 Survey: 100 people, 40 under age 30, 28 of them like coffee. Find:

$$P(\text{likes coffee} | \text{under 30})$$

$$= \frac{28}{40} = 0.70$$

Condition on under-30 group (40 people). Of those, 28 like coffee: $28/40 = 0.70$.

7 Find $P(A \cap B)$:

$$P(B|A) = 0.70, \quad P(A) = 0.30$$

$$= P(A \cap B) = 0.70 \times 0.30 = 0.21$$

Multiplication rule: $P(A \cap B) = P(B|A) \cdot P(A) = 0.70 \times 0.30 = 0.21$.

8 Test independence: does $P(A \cap B) = P(A) \cdot P(B)$?

$$P(A) = 0.50, \quad P(B) = 0.40, \quad P(A \cap B) = 0.25$$

$$= 0.50 \times 0.40 = 0.20 \neq 0.25 \Rightarrow \text{Dependent}$$

If $P(A \cap B) \neq P(A) \cdot P(B)$, events are dependent. $0.25 \neq 0.20$, so dependent.