Statistics: Chi-Square Test for Independence

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DIRECTIONS *Use the chi-square formulas: E = (row total × col total) / grand total; $\chi^2 = \Sigma(O-E)^2/E$; df = (rows–1)(cols–1). If χ^2 > critical value, reject H■ and conclude the variables are not independent.*

1 Find the degrees of freedom for a 2 \times 3 contingency table:

$$df = (r-1)(c-1)$$

Answer: ______________________

2 Find the expected frequency E:

$$E = \frac{\text{row total} \times \text{col total}}{\text{grand total}} = \frac{40 \times 30}{100}$$

Answer: ______________________

3 Compute the \chi^{2} contribution for one cell:

$$O = 15, \quad E = 12$$

Answer: ______________________

4 Find the degrees of freedom for a 3 \times 4 contingency table:

$$df = (r-1)(c-1)$$

Answer: ______________________

5 Find the expected frequency E:

$$E = \frac{60 \times 50}{200}$$

Answer: ______________________

6 Compute total \chi^{2} for these four cells:

$$\frac{(12-10)^2}{10} + \frac{(8-10)^2}{10} + \frac{(18-20)^2}{20} + \frac{(22-20)^2}{20}$$

Answer: ______________________

7 At \alpha = 0.05, df = 2, critical value = 5.991. Make a decision:

$$\chi^2 = 7.32 > 5.991$$

Answer: ______________________

8 Compute the \chi^{2} contribution for one cell:

$$O = 25, \quad E = 20$$

Answer: ______________________

Answer Key & Solutions

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TEACHER NOTES *E = (row total × col total) / grand total. χ² = sum of $(O–E)^2/E$ over all cells. df = (r–1)(c–1). Reject H0 if χ² > χ²_critical (from table at α and df). All E values should be ≥ 5 for the test to be valid.*

1 Find the degrees of freedom for a 2 \times 3 contingency table:

$$df = (r-1)(c-1)$$

= $$df = (2-1)(3-1) = 2$$

df = (rows–1)(cols–1) = (1)(2) = 2.

2 Find the expected frequency E:

$$E = \frac{\text{row total} \times \text{col total}}{\text{grand total}} = \frac{40 \times 30}{100}$$

= $$E = 12$$

E = (row total × col total) / grand total = 1200/100 = 12.

3 Compute the \chi^{2} contribution for one cell:

$$O = 15, \quad E = 12$$

= $$\frac{(15-12)^2}{12} = \frac{9}{12} = 0.75$$

Each cell contributes $(O–E)^2/E$. Here $(3)^2/12$ = 9/12 = 0.75.

4 Find the degrees of freedom for a 3 \times 4 contingency table:

$$df = (r-1)(c-1)$$

= $$df = (3-1)(4-1) = 6$$

df = (2)(3) = 6. More cells → more degrees of freedom.

5 Find the expected frequency E:

$$E = \frac{60 \times 50}{200}$$

= $$E = \frac{3000}{200} = 15$$

E = (row total × col total) / grand total = 3000/200 = 15.

6 Compute total \chi^{2} for these four cells:

$$\frac{(12-10)^2}{10} + \frac{(8-10)^2}{10} + \frac{(18-20)^2}{20} + \frac{(22-20)^2}{20}$$

= $$0.40 + 0.40 + 0.20 + 0.20 = 1.20$$

Sum all cell contributions: 4/10 + 4/10 + 4/20 + 4/20 = 0.4+0.4+0.2+0.2 = 1.20.

7 At \alpha = 0.05, df = 2, critical value = 5.991. Make a decision:

$$\chi^2 = 7.32 > 5.991$$

= $\text{Reject } H_0 \text{ — not independent}$

Since χ^2 = 7.32 exceeds the critical value 5.991, reject H0 at α = 0.05.

8 Compute the \chi^{2} contribution for one cell:

$$O = 25, \quad E = 20$$

= $$\frac{(25-20)^2}{20} = \frac{25}{20} = 1.25$$

$(O–E)^2/E = (5)^2/20$ = 25/20 = 1.25.