Z-Scores and the Standard Normal Distribution

Statistics Worksheet · Grades 10–12

Name: ______________________________ Date: ____________________

Learning Objectives

- Calculate the z-score of a raw data value using the formula z = (x – μ) / σ
- Interpret the meaning of a z-score within a normal distribution
- Compare scores from different normal distributions using standardized z-scores

Problems

1. A normal distribution has a mean of 50 and a standard deviation of 10. Write the statistical notation for this distribution.

$$N(\mu, \sigma)$$

2. A normal distribution has a mean of 75 and a standard deviation of 8. Find the z-score for a raw score of 83.

$$z = \frac{x - \mu}{\sigma}$$

3. A normal distribution has a mean of 100 and a standard deviation of 15. Find the z-score for a raw score of 70.

$$z = \frac{x - \mu}{\sigma}$$

4. The heights of adult males are normally distributed with a mean of 70 inches and a standard deviation of 3 inches. Find the z-score for a male who is 76 inches tall. Then locate whether this value falls above or below the mean.

$$z = \frac{x - \mu}{\sigma}$$

5. The exam scores of a class follow a normal distribution with a mean of 72 and a standard deviation of 9. Fill in the missing z-scores in the table below for each raw score.

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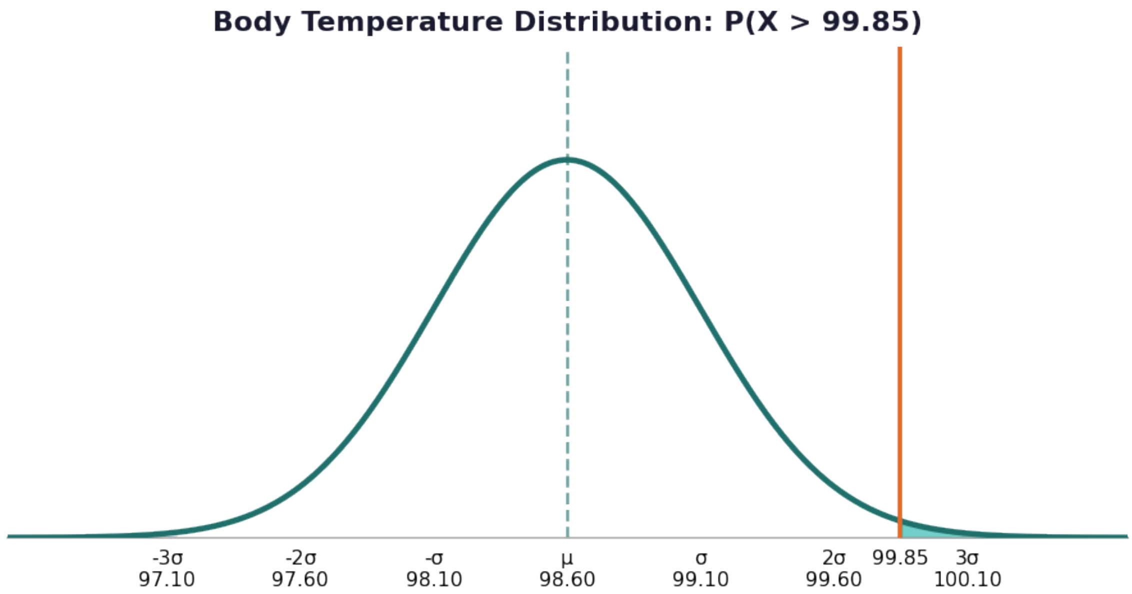
Raw Score (x)	Mean (μ)	Std Dev (σ)	Z-Score
63	72	9	
81	72	9	
72	72	9	
54	72	9	

6. Body temperatures are normally distributed with a mean of 98.6 degrees Fahrenheit and a standard deviation of 0.5 degrees. Find the z-score for a body temperature of 99.85 degrees. On the standard normal curve shown, shade the region to the right of this z-score.



7. Maria scored 88 on a science test that had a mean of 78 and a standard deviation of 8. John scored 74 on a different science test that had a mean of 65 and a standard deviation of 6. Assuming both tests measure the same ability, who performed better relative to their class?

$$z = \frac{x - \mu}{\sigma}$$

8. The annual rainfall in a city is normally distributed with a mean of 45 inches and a standard deviation of 7 inches. Find the probability that a randomly selected year had rainfall between 38 inches and 59 inches. Use the standard normal distribution and z-scores to set up the problem.

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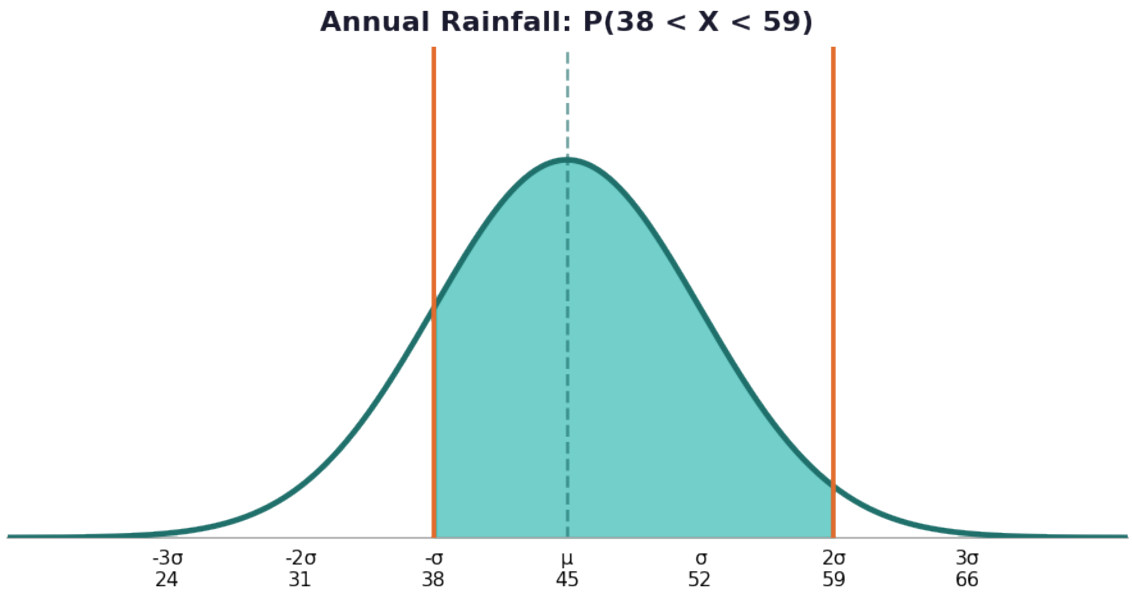
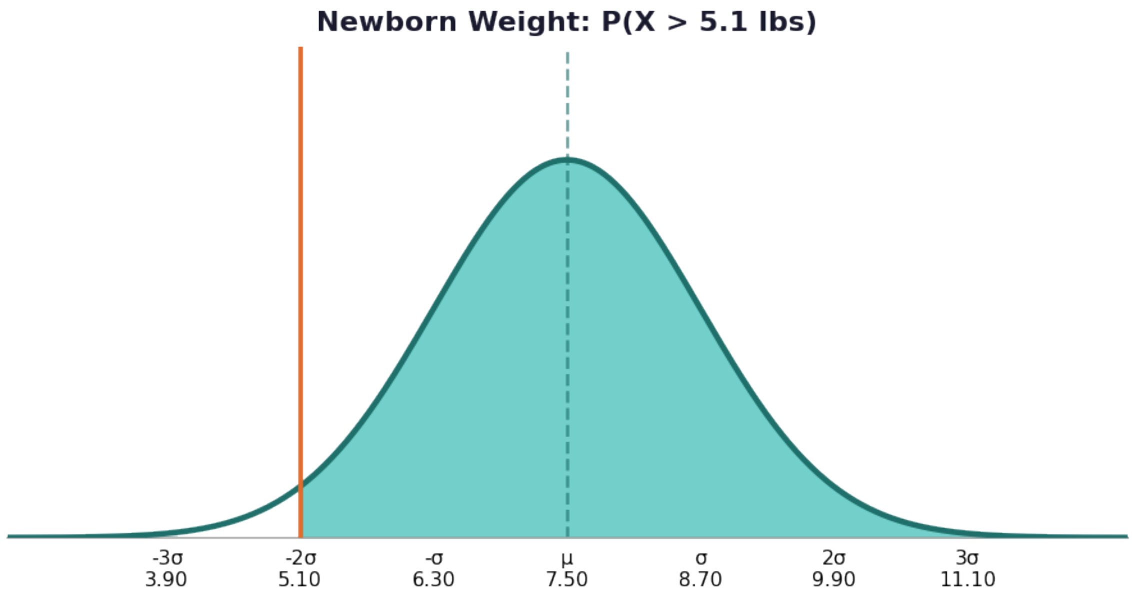


9. Three students took three different standardized tests. Use the z-score formula to find each student's standardized score, then rank them from highest to lowest performance relative to their test group.

Student	Score (x)	Mean (μ)	Std Dev (σ)	Z-Score	Rank
Alex	540	500	80		
Brianna	30	21	6		
Carlos	610	500	100		

10. The weights of newborn babies are normally distributed with a mean of 7.5 pounds and a standard deviation of 1.2 pounds. A hospital considers a baby to be underweight if its weight falls more than 2 standard deviations below the mean. (a) What is the minimum z-score for a baby NOT to be considered underweight? (b) What is the actual weight in pounds that corresponds to this z-score? (c) Find the probability that a randomly selected newborn is NOT underweight, meaning its weight is above this cutoff. Show the shaded region on the normal curve.


Newborn Weight: P(X > 5.1 lbs)
-3σ
3.90
-2σ
5.10
-σ
6.30
μ
7.50
σ
8.70
2σ
9.90
3σ
11.10

Z-Scores and the Standard Normal Distribution — Answer Key

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Answer Key

1. Answer: N(50, 10)

- The notation for a normal distribution is $N(\mu, \sigma)$
- Here $\mu = 50$ and $\sigma = 10$
- So the notation is N(50, 10)

2. Answer: z = 1.0

- Identify the values: $x = 83$, $\mu = 75$, $\sigma = 8$
- Apply the z-score formula: $z = (83 - 75) / 8$
- $z = 8 / 8 = 1.0$

3. Answer: z = –2.0

- Identify the values: $x = 70$, $\mu = 100$, $\sigma = 15$
- Apply the z-score formula: $z = (70 - 100) / 15$
- $z = -30 / 15 = -2.0$
- The negative z-score means the raw score is 2 standard deviations below the mean

4. Answer: z = 2.0; above the mean

- Identify the values: $x = 76$, $\mu = 70$, $\sigma = 3$
- Apply the z-score formula: $z = (76 - 70) / 3$
- $z = 6 / 3 = 2.0$
- Since z is positive, this height is 2 standard deviations above the mean

5. Answer: z = –1.0, 1.0, 0.0, –2.0

Raw Score (x)	Mean (μ)	Std Dev (σ)	Z-Score
63	72	9	−1.0
81	72	9	1.0
72	72	9	0.0
54	72	9	−2.0

- Use $z = (x - \mu) / \sigma$ for each row
- Row 1: $z = (63 - 72) / 9 = -9 / 9 = -1.0$
- Row 2: $z = (81 - 72) / 9 = 9 / 9 = 1.0$
- Row 3: $z = (72 - 72) / 9 = 0 / 9 = 0.0$ (the mean always has $z = 0$)
- Row 4: $z = (54 - 72) / 9 = -18 / 9 = -2.0$

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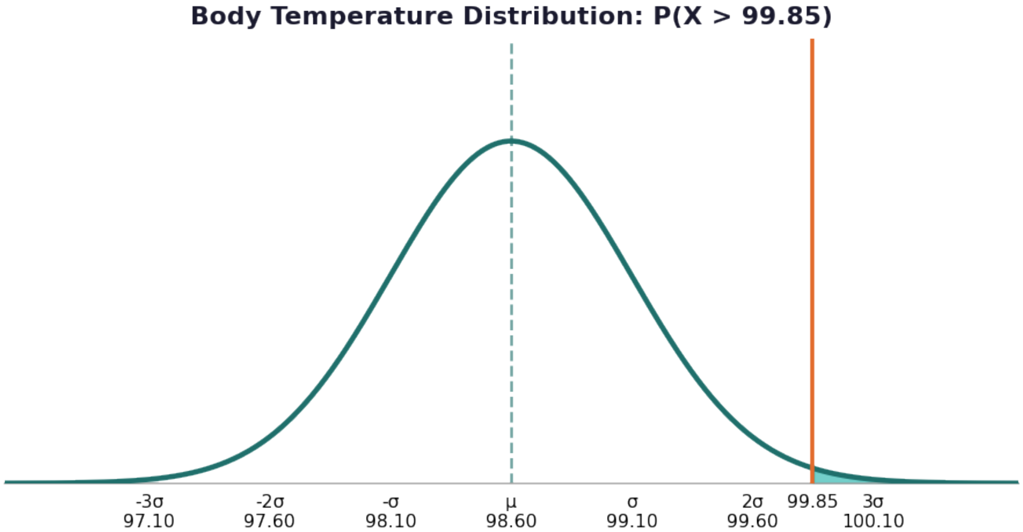
6. Answer: z = 2.5; P ≈ 0.0062 (0.62%)

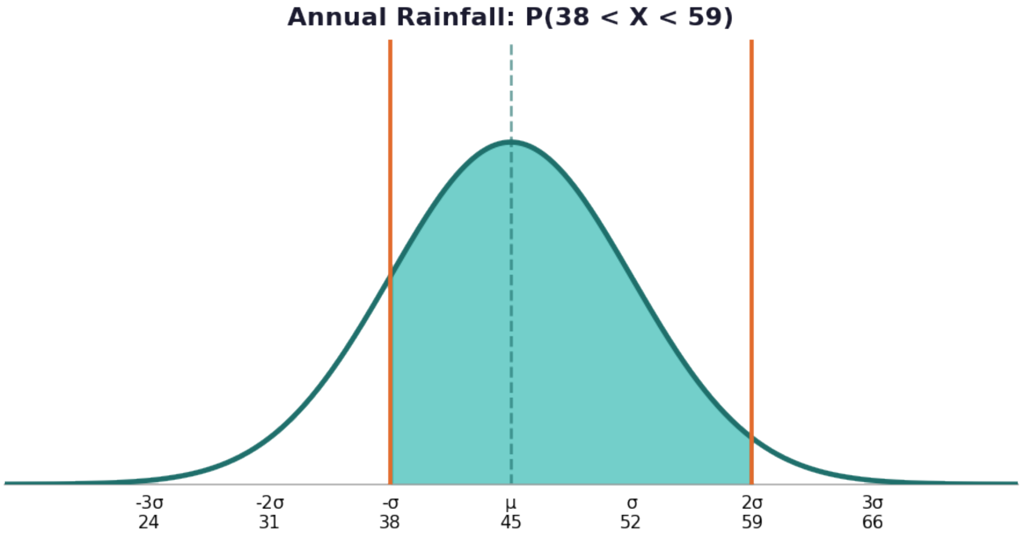


- Identify values: x = 99.85, μ = 98.6, σ = 0.5
- Apply z = (99.85 – 98.6) / 0.5
- z = 1.25 / 0.5 = 2.5
- A z-score of 2.5 is in the far right tail of the standard normal distribution
- Using a z-table, P(Z > 2.5) ≈ 0.0062, meaning only about 0.62% of people have this temperature or higher

7. Answer: John performed better (z = 1.5 vs Maria's z = 1.25)

- Find Maria's z-score: z = (88 – 78) / 8 = 10 / 8 = 1.25
- Find John's z-score: z = (74 – 65) / 6 = 9 / 6 = 1.5
- John's z-score of 1.5 is higher than Maria's z-score of 1.25
- Therefore, John performed better relative to his class, even though his raw score was lower

8. Answer: P ≈ 0.8368 (83.68%)



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- Find z-score for x = 38: z■ = (38 – 45) / 7 = –7 / 7 = –1.0
- Find z-score for x = 59: z■ = (59 – 45) / 7 = 14 / 7 = 2.0
- Find P(–1.0 < Z < 2.0) using z-table
- P(Z < 2.0) ≈ 0.9772 and P(Z < –1.0) ≈ 0.1587
- P(–1.0 < Z < 2.0) = 0.9772 – 0.1587 = 0.8185 ≈ 83.85%

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9. Answer: Brianna (z=1.5), Carlos (z=1.1), Alex (z=0.5); Rank: 1st Brianna, 2nd Carlos, 3rd Alex

Student	Score (x)	Mean (μ)	Std Dev (σ)	Z-Score	Rank
Alex	540	500	80	0.5	3rd
Brianna	30	21	6	1.5	1st
Carlos	610	500	100	1.1	2nd

- Alex: z = (540 – 500) / 80 = 40 / 80 = 0.5
- Brianna: z = (30 – 21) / 6 = 9 / 6 = 1.5
- Carlos: z = (610 – 500) / 100 = 110 / 100 = 1.1
- Ranking by z-score: Brianna (1.5) > Carlos (1.1) > Alex (0.5)
- Even though Carlos had the highest raw score, Brianna performed best relative to her test group

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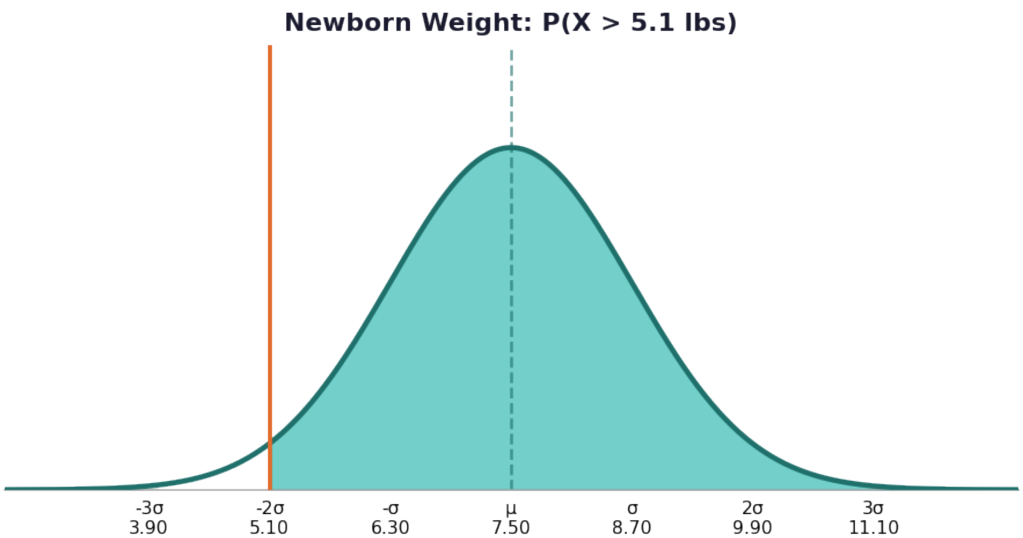
10. Answer: (a) z = –2.0; (b) x = 5.1 lbs; (c) P ≈ 0.9772 (97.72%)



- (a) The cutoff is exactly 2 standard deviations below the mean, so z = –2.0
- (b) Solve for x using x = μ + z·σ: x = 7.5 + (–2.0)(1.2) = 7.5 – 2.4 = 5.1 pounds
- (c) We need P(X > 5.1), which equals P(Z > –2.0)
- Using a z-table: P(Z < –2.0) ≈ 0.0228
- P(Z > –2.0) = 1 – 0.0228 = 0.9772
- About 97.72% of newborns would NOT be classified as underweight

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