The Empirical Rule (68–95–99.7 Rule)

Statistics Worksheet · Grade 10–12

Name: ________________________________ Date: __________________

Learning Objectives

- Sketch a normal distribution and label values at each standard deviation using the mean and standard deviation
- Apply the 68–95–99.7 rule to find the percentage of data within 1, 2, or 3 standard deviations of the mean
- Use symmetry and algebra to find the proportion of data between any two values on a normal distribution

Problems

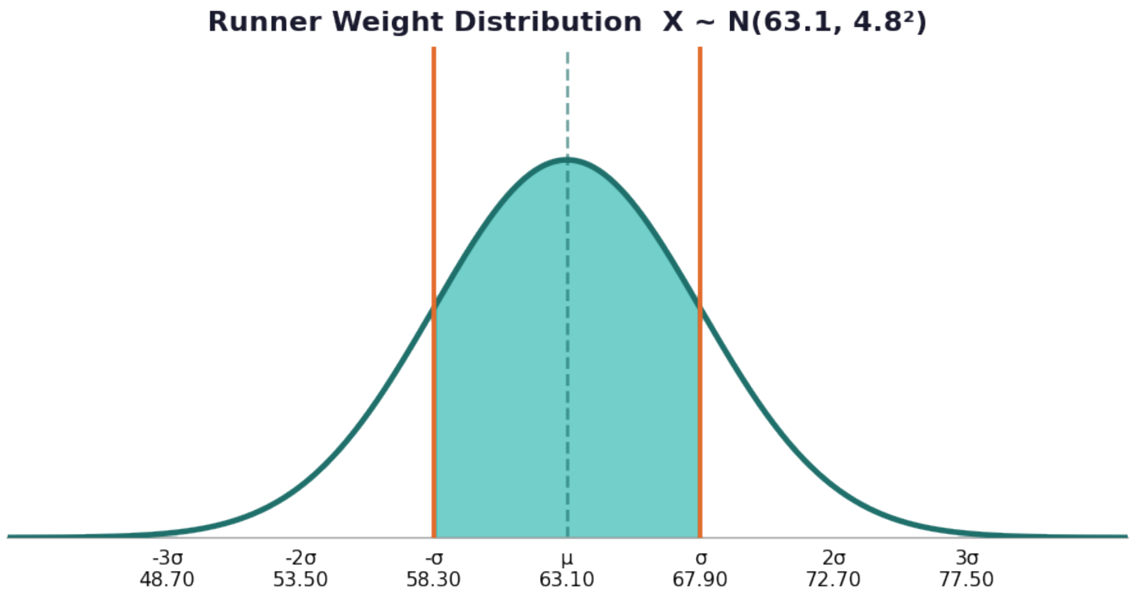
1. The body weights of a group of runners are normally distributed with a mean of 63.1 kg and a standard deviation of 4.8 kg. What value is exactly one standard deviation above the mean?

$$\mu = 63.1, \quad \sigma = 4.8$$

2. Using the same runner weight distribution (mean = 63.1 kg, standard deviation = 4.8 kg), list all six values that mark 1, 2, and 3 standard deviations on both sides of the mean.

$$\mu = 63.1, \quad \sigma = 4.8$$

3. A normal distribution has a mean of 63.1 kg and a standard deviation of 4.8 kg. Sketch and shade the region that contains 68% of the data according to the empirical rule. Identify the two boundary values.



4. Using the empirical rule with mean = 63.1 kg and standard deviation = 4.8 kg, what percentage of runners have a body weight between 53.5 kg and 72.7 kg?

$$P(53.5 < X < 72.7)$$

5. Using the runner weight distribution (mean = 63.1 kg, standard deviation = 4.8 kg), what percentage of runners have a body weight between 48.7 kg and 77.5 kg?

$$P(48.7 < X < 77.5)$$

6. Using the runner weight distribution (mean = 63.1 kg, standard deviation = 4.8 kg), find the proportion of runners whose body weight is between 63.1 kg and 72.7 kg. Hint: use half of the appropriate empirical rule percentage.

$$P(63.1 < X < 72.7)$$

7. Using the runner weight distribution (mean = 63.1 kg, standard deviation = 4.8 kg), find the proportion of runners whose body weight is between 48.7 kg and 67.9 kg.

$$P(48.7 < X < 67.9)$$

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8. The heights of adult women in a city are normally distributed with a mean of 162 cm and a standard deviation of 6 cm. Use the empirical rule to find the percentage of women with heights between 150 cm and 174 cm.

$$P(150 < X < 174), \quad \mu = 162, \quad \sigma = 6$$

9. The scores on a standardized test are normally distributed with a mean of 500 and a standard deviation of 50. Using the empirical rule, find the probability that a randomly selected student scored above 600.

$$P(X > 600), \quad \mu = 500, \quad \sigma = 50$$

10. The daily high temperatures in a city during summer are normally distributed with a mean of 88 degrees Fahrenheit and a standard deviation of 4 degrees. Using the empirical rule, find the percentage of days with a high temperature between 80 degrees and 92 degrees Fahrenheit.

$$P(80 < X < 92), \quad \mu = 88, \quad \sigma = 4$$

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The Empirical Rule (68–95–99.7 Rule) — Answer Key

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Answer Key

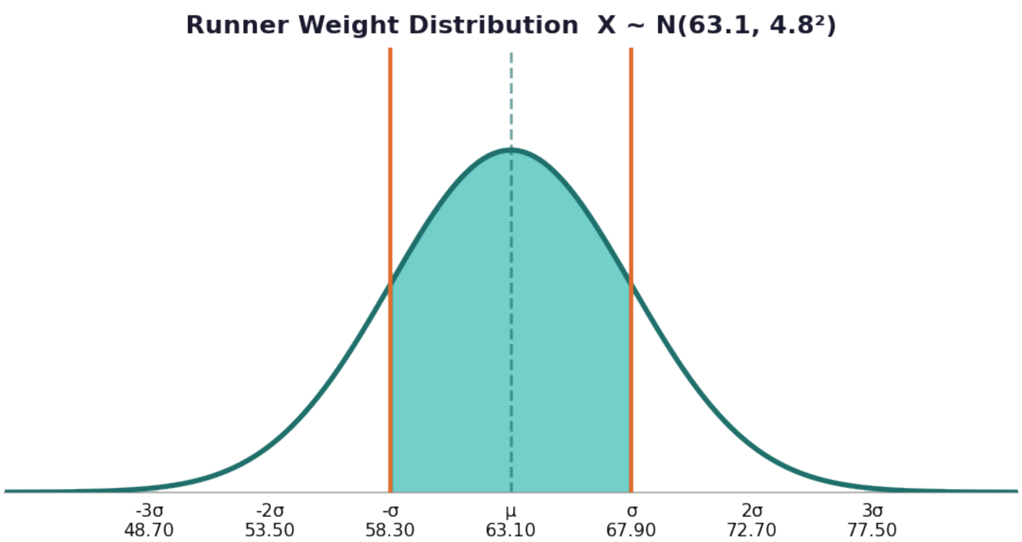
1. Answer: 67.9 kg

- One standard deviation above the mean = μ + σ
- 63.1 + 4.8 = 67.9 kg

2. Answer: 48.7, 53.5, 58.3, 67.9, 72.7, 77.5

- Left side: 63.1 – 4.8 = 58.3; 63.1 – 9.6 = 53.5; 63.1 – 14.4 = 48.7
- Right side: 63.1 + 4.8 = 67.9; 63.1 + 9.6 = 72.7; 63.1 + 14.4 = 77.5

3. Answer: 58.3 kg to 67.9 kg (68% of runners)



- The 68% region spans one standard deviation on each side of the mean
- Lower boundary: 63.1 – 4.8 = 58.3 kg
- Upper boundary: 63.1 + 4.8 = 67.9 kg

4. Answer: 95%

- 53.5 = 63.1 – 2(4.8), so 53.5 is 2 standard deviations below the mean
- 72.7 = 63.1 + 2(4.8), so 72.7 is 2 standard deviations above the mean
- By the empirical rule, 95% of data falls within 2 standard deviations of the mean

5. Answer: 99.7%

- 48.7 = 63.1 – 3(4.8), so 48.7 is 3 standard deviations below the mean
- 77.5 = 63.1 + 3(4.8), so 77.5 is 3 standard deviations above the mean

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- By the empirical rule, 99.7% of data falls within 3 standard deviations of the mean

6. Answer: 47.5%

- 72.7 is 2 standard deviations above the mean; 63.1 is the mean
- The full 2-standard-deviation range covers 95% of data (both sides)
- By symmetry, from the mean to 2σ above = 95% ÷ 2 = 47.5%

7. Answer: 83.85%

- 48.7 is 3 standard deviations below the mean → from 48.7 to mean = 99.7 ÷ 2 = 49.85%
- 67.9 is 1 standard deviation above the mean → from mean to 67.9 = 68 ÷ 2 = 34%
- Total area = 49.85% + 34% = 83.85%

8. Answer: 95%

- 150 = 162 – 2(6) → 2 standard deviations below the mean
- 174 = 162 + 2(6) → 2 standard deviations above the mean
- By the empirical rule, 95% of women have heights between 150 cm and 174 cm

9. Answer: 2.5%

- 600 = 500 + 2(50) → 600 is 2 standard deviations above the mean
- By the empirical rule, 95% of scores fall between 400 and 600
- The remaining 5% is split equally into two tails: 2.5% below 400 and 2.5% above 600
- Therefore $P(X > 600) = 2.5\%$

10. Answer: 81.5%

- 80 = 88 – 2(4) → 80 is 2 standard deviations below the mean; from 80 to mean = 95 ÷ 2 = 47.5%
- 92 = 88 + 1(4) → 92 is 1 standard deviation above the mean; from mean to 92 = 68 ÷ 2 = 34%
- Total area between 80 and 92 = 47.5% + 34% = 81.5%