Normal Distribution & Probability

Statistics Worksheet · Grades 11–12

Name: ______________________________ Date: __________________

Learning Objectives

- Convert raw scores to z-scores using the standardization formula
- Find probabilities using the standard normal distribution (normalcdf)
- Interpret left-tail, right-tail, and between-values probabilities in context

Problems

1. The average height of students at Lincoln High is 67 inches with a standard deviation of 2.5 inches. Find the z-score for a student who is 72 inches tall.

$$z = \frac{x - \mu}{\sigma} = \frac{72 - 67}{2.5}$$

2. Using the same distribution (mean = 67, standard deviation = 2.5), find the z-score for a student who is 60 inches tall.

$$z = \frac{60 - 67}{2.5}$$

3. A normally distributed data set has a mean of 50 and a standard deviation of 8. Match each raw score to its correct z-score by completing the table below.

Raw Score (x)	z-score
34	
50	
58	
66	

4. The heights of students at Barstow High are normally distributed with a mean of 67 inches and a standard deviation of 2.5 inches. What is the probability that a randomly selected student is shorter than 60 inches? Use the normalcdf function with the standardized z-score.

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$$P(X < 60) = P\left(Z < \frac{60-67}{2.5}\right) = P(Z < -2.8)$$

5. Explain why the probability that a student is EXACTLY 73 inches tall — in a continuous normal distribution — is equal to zero. Then find the z-score for 73 inches using mean = 67 and standard deviation = 2.5.

$$z = \frac{73-67}{2.5}$$

6. Using mean = 67 inches and standard deviation = 2.5 inches, find the probability that a student is taller than 71 inches. Shade the correct region and write the normalcdf syntax you would enter in a TI-84.

$$P(X > 71) = P\left(Z > \frac{71-67}{2.5}\right) = P(Z > 1.6)$$

7. The heights at Barstow High follow a normal distribution with mean 67 inches and standard deviation 2.5 inches. Find the probability that a randomly selected student has a height between 61 inches and 71 inches.

$$P(61 < X < 71) = P\left(\frac{61-67}{2.5} < Z < \frac{71-67}{2.5}\right)$$

8. A student claims that about half the students at Barstow High are shorter than 67 inches. Using the properties of a normal distribution with mean 67 and standard deviation 2.5, confirm or refute this claim by calculating P(X less than 67).

$$P(X < 67) = P\left(Z < \frac{67-67}{2.5}\right) = P(Z < 0)$$

9. Test scores at a school are normally distributed with a mean of 78 and a standard deviation of 6. Find the probability that a randomly selected student scored between 70 and 88. Also identify how many standard deviations each boundary is from the mean.

$$P(70 < X < 88) = P\left(\frac{70-78}{6} < Z < \frac{88-78}{6}\right)$$

10. Weights of adult males in a city are normally distributed with a mean of 180 pounds and a standard deviation of 15 pounds. A health clinic defines 'underweight' as below 150 pounds and 'overweight' as above 210 pounds. Find (a) the probability a randomly selected male is underweight, (b) the probability he is overweight, and (c) the probability he falls in the healthy range between 150 and 210 pounds.

$$P(X < 150), \quad P(X > 210), \quad P(150 < X < 210)$$

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Normal Distribution & Probability — Answer Key

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Answer Key

1. Answer: z = 2.0

- Identify the values: $x = 72$, $\mu = 67$, $\sigma = 2.5$
- Substitute into the z-score formula: $z = (72 - 67) / 2.5$
- Simplify: $z = 5 / 2.5 = 2.0$

2. Answer: z = -2.8

- Identify the values: $x = 60$, $\mu = 67$, $\sigma = 2.5$
- Substitute: $z = (60 - 67) / 2.5 = -7 / 2.5$
- Simplify: $z = -2.8$

3. Answer: See completed table

Raw Score (x)	z-score
34	-2.0
50	0.0
58	1.0
66	2.0

- Use $z = (x - \mu) / \sigma$ for each value with $\mu = 50$ and $\sigma = 8$
- $z = (34 - 50) / 8 = -16 / 8 = -2.0$
- $z = (50 - 50) / 8 = 0 / 8 = 0.0$
- $z = (58 - 50) / 8 = 8 / 8 = 1.0$
- $z = (66 - 50) / 8 = 16 / 8 = 2.0$

4. Answer: $P(X < 60) \approx 0.0026$ or about 0.26%

- Compute $z = (60 - 67) / 2.5 = -2.8$
- Use normalcdf(–1E99, –2.8, 0, 1) on the TI-84
- The calculator returns approximately 0.0026
- Interpret: about 0.26% of students are shorter than 60 inches

5. Answer: $P(X = 73) = 0$; $z = 2.4$

- In a continuous distribution, the probability of any single exact value is 0 because a single point has no area under the curve
- Compute $z = (73 - 67) / 2.5 = 6 / 2.5 = 2.4$
- Although $z = 2.4$, $P(X = 73) = 0$ for a continuous distribution

6. Answer: $P(X > 71) \approx 0.0548$ or about 5.48%

- Compute $z = (71 - 67) / 2.5 = 4 / 2.5 = 1.6$

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- Since we want $P(Z > 1.6)$, the shaded region is to the right
- TI-84 syntax: normalcdf(1.6, 1E99, 0, 1)
- Result ≈ 0.0548, so about 5.48% of students are taller than 71 inches

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7. Answer: $P(61 < X < 71) \approx 0.9104$ or about 91.04%

- Compute lower z: z■ = (61 – 67) / 2.5 = –2.4
- Compute upper z: z■ = (71 – 67) / 2.5 = 1.6
- TI-84 syntax: normalcdf(–2.4, 1.6, 0, 1)
- Result ≈ 0.9104, meaning about 91.04% of students have heights in this range

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8. Answer: $P(X < 67) = 0.5000$ — the claim is correct

- Compute $z = (67 - 67) / 2.5 = 0$
- In a normal distribution, $P(Z < 0) = 0.5000$ exactly
- The mean is the center of the distribution, so exactly 50% of values fall below it
- The student's claim is correct

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9. Answer: $P(70 < X < 88) \approx 0.8164$ or about 81.64%

- Compute lower z: z■ = (70 – 78) / 6 = –8 / 6 ≈ –1.33 (about 1.33 standard deviations below the mean)
- Compute upper z: z■ = (88 – 78) / 6 = 10 / 6 ≈ 1.67 (about 1.67 standard deviations above the mean)
- TI-84 syntax: normalcdf(–1.33, 1.67, 0, 1)
- Result ≈ 0.8164, so about 81.64% of students scored between 70 and 88

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10. Answer: (a) ≈ 0.0228 or 2.28%; (b) ≈ 0.0228 or 2.28%; (c) ≈ 0.9545 or 95.45%

- Compute z for 150: $z = (150 - 180) / 15 = -30 / 15 = -2.0$
- Compute z for 210: $z = (210 - 180) / 15 = 30 / 15 = 2.0$
- (a) $P(X < 150)$ = normalcdf(–1E99, –2.0, 0, 1) ≈ 0.0228
- (b) $P(X > 210)$ = normalcdf(2.0, 1E99, 0, 1) ≈ 0.0228
- (c) $P(150 < X < 210)$ = normalcdf(–2.0, 2.0, 0, 1) ≈ 0.9545
- Note: This confirms the empirical rule — approximately 95.45% of data falls within 2 standard deviations of the mean

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