Normal Distribution & Sampling: Using the Z-Table

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Name: ______________________________ Date: __________________

Learning Objectives

- Calculate z-scores for sampling distributions using the formula z = (x■ – μ) / (σ / √n)
- Use the Z-table to find probabilities (areas under the normal curve) for given z-scores
- Interpret probability results in context, including less than, greater than, and between scenarios

Problems

1. The GPAs of students at a university are approximately normally distributed with a mean of 3.05 and a standard deviation of 0.29. A random sample of 20 students is selected. What is the standard error of the mean (the denominator of the z-score formula) for this sample? Round to four decimal places.

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{0.29}{\sqrt{20}}$$

2. Using the same university GPA distribution (mean = 3.05, standard deviation = 0.29, sample size = 20), calculate the z-score for a sample mean of 2.98. Round to two decimal places.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{2.98 - 3.05}{0.29/\sqrt{20}}$$

3. Using a Z-table and the z-score of –1.08 found above, find the probability that the sample mean GPA of 20 students is less than or equal to 2.98. Express your answer as a percentage.

$$P(\bar{x} \leq 2.98) = P(z \leq -1.08)$$

4. Using the same GPA distribution (mean = 3.05, standard deviation = 0.29, sample size = 20), calculate the z-score for a sample mean of 3.01. Round to two decimal places.

$$z = \frac{3.01 - 3.05}{0.29/\sqrt{20}}$$

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5. Using the z-score of –0.62, find the probability that the sample mean GPA of 20 students is greater than 3.01. Use the Z-table and recall that P(z > a) = 1 – P(z ≤ a). Express your answer as a percentage.

$$P(\bar{x} > 3.01) = 1 - P(z \leq -0.62)$$

6. The table below shows z-scores and their corresponding Z-table values. Fill in the two missing probability values for z = –1.45 and z = 0.78.

z-score	P(Z ≤ z)
-1.45	
-0.62	0.2676
-1.08	0.1401
0.78	

7. Using the GPA distribution (mean = 3.05, standard deviation = 0.29, sample size = 20), find the probability that the sample mean GPA is between 2.90 and 3.07. First calculate both z-scores, then use the Z-table. Express your answer as a percentage.

$$P(2.90 \leq \bar{x} \leq 3.07) = P\left(\frac{2.90 - 3.05}{0.29/\sqrt{20}} \leq z \leq \frac{3.07 - 3.05}{0.29/\sqrt{20}}\right)$$

8. The heights of adult males are normally distributed with a mean of 70 inches and a standard deviation of 3 inches. A random sample of 36 males is taken. Find the probability that the sample mean height is between 69.5 inches and 70.8 inches. Show all steps including both z-scores. Express your answer as a percentage.

$$P(69.5 \leq \bar{x} \leq 70.8) = P\left(\frac{69.5 - 70}{3/\sqrt{36}} \leq z \leq \frac{70.8 - 70}{3/\sqrt{36}}\right)$$

9. A standardized exam has scores that are normally distributed with a mean of 500 and a standard deviation of 100. A random sample of 25 students is selected. Find the probability that the sample mean score is greater than 530. Show all work and express your answer as a percentage.

$$P(\bar{x} > 530) = 1 - P\left(z \leq \frac{530 - 500}{100/\sqrt{25}}\right)$$

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10. A factory fills bottles with a mean volume of 16 oz and a standard deviation of 0.5 oz. Quality control randomly samples 64 bottles. (a) Find the probability that the sample mean volume is less than 15.88 oz. (b) Find the probability that the sample mean volume is between 15.95 oz and 16.10 oz. Express both answers as percentages.

$P(\bar{x} < 15.88)$ and $P(15.95 \leq \bar{x} \leq 16.10)$

Normal Distribution & Sampling: Using the Z-Table — Answer Key

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Answer Key

1. Answer: SE ≈ 0.0648

- Identify $\sigma = 0.29$ and $n = 20$.
- Compute $\sqrt{20} \approx 4.4721$.
- Divide: $0.29 \div 4.4721 \approx 0.0648$.

2. Answer: z ≈ –1.08

- Compute the numerator: $2.98 - 3.05 = -0.07$.
- Compute the denominator (standard error): $0.29 / \sqrt{20} \approx 0.0648$.
- Divide: $-0.07 \div 0.0648 \approx -1.08$.

3. Answer: P ≈ 14.01%

- Locate –1.0 in the left column of the negative Z-table.
- Move across to the column for 0.08.
- The table value at $z = -1.08$ is 0.1401.
- Convert to percentage: 14.01%.

4. Answer: z ≈ –0.62

- Compute the numerator: $3.01 - 3.05 = -0.04$.
- Standard error = $0.29 / \sqrt{20} \approx 0.0648$.
- Divide: $-0.04 \div 0.0648 \approx -0.62$.

5. Answer: P ≈ 73.24%

- Look up $z = -0.62$ in the negative Z-table: $P(z \leq -0.62) \approx 0.2676$.
- Subtract from 1: $1 - 0.2676 = 0.7324$.
- Convert to percentage: 73.24%.

6. Answer: P(Z ≤ –1.45) ≈ 0.0735; P(Z ≤ 0.78) ≈ 0.7823

z-score	P(Z ≤ z)
-1.45	0.0735
-0.62	0.2676
-1.08	0.1401
0.78	0.7823

- For $z = -1.45$: locate row –1.4 and column 0.05 in the negative Z-table → 0.0735.
- For $z = 0.78$: locate row 0.7 and column 0.08 in the positive Z-table → 0.7823.

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7. Answer: P ≈ 61.06%

- Standard error = 0.29 / √20 ≈ 0.0648.
- z■ = (2.90 – 3.05) / 0.0648 = –0.15 / 0.0648 ≈ –2.31.
- z■ = (3.07 – 3.05) / 0.0648 = 0.02 / 0.0648 ≈ 0.31.
- P(z ≤ –2.31) ≈ 0.0104; P(z ≤ 0.31) ≈ 0.6217.
- P(–2.31 ≤ z ≤ 0.31) = 0.6217 – 0.0104 = 0.6113 ≈ 61.13%.

8. Answer: P ≈ 84.13%

- Standard error = 3 / √36 = 3 / 6 = 0.5.
- z■ = (69.5 – 70) / 0.5 = –0.5 / 0.5 = –1.00.
- z■ = (70.8 – 70) / 0.5 = 0.8 / 0.5 = 1.60.
- P(z ≤ –1.00) ≈ 0.1587; P(z ≤ 1.60) ≈ 0.9452.
- P(–1.00 ≤ z ≤ 1.60) = 0.9452 – 0.1587 = 0.7865 ≈ 78.65%.

9. Answer: P ≈ 6.68%

- Standard error = 100 / √25 = 100 / 5 = 20.
- z = (530 – 500) / 20 = 30 / 20 = 1.50.
- P(z ≤ 1.50) ≈ 0.9332 from the positive Z-table.
- P(z > 1.50) = 1 – 0.9332 = 0.0668.
- Convert to percentage: 6.68%.

10. Answer: (a) P ≈ 2.84% (b) P ≈ 54.67%

- Standard error = 0.5 / √64 = 0.5 / 8 = 0.0625.
- Part (a): z = (15.88 – 16) / 0.0625 = –0.12 / 0.0625 = –1.92.
- P(z ≤ –1.92) ≈ 0.0274 → about 2.74%.
- Part (b): z■ = (15.95 – 16) / 0.0625 = –0.05 / 0.0625 = –0.80.
- z■ = (16.10 – 16) / 0.0625 = 0.10 / 0.0625 = 1.60.
- P(z ≤ –0.80) ≈ 0.2119; P(z ≤ 1.60) ≈ 0.9452.
- P(–0.80 ≤ z ≤ 1.60) = 0.9452 – 0.2119 = 0.7333 → about 73.33%.

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