Discrete & Continuous Random Variables · Normal Distribution

Statistics Worksheet · Grade 11–12

Name: ______________________________ Date: __________________

Learning Objectives

- Distinguish between discrete and continuous random variables and give real-world examples of each
- Interpret and apply the z-score formula to find probabilities and raw scores under a normal distribution
- Use the standard normal table to find areas, percentiles, and cutoff scores for real-world problems

Problems

1. A fair coin is flipped 4 times. Let X be the number of heads. List all possible values of X and identify whether X is a discrete or continuous random variable.

$$X = \{0, 1, 2, 3, 4\}$$

2. For each scenario below, identify whether the random variable is discrete (D) or continuous (C). Fill in the table.

Scenario	D or C?
Number of students absent from school each day	
The time it takes a runner to finish a 5K race	
The number of cars in a parking lot	
The weight of a newborn baby	
The number of tails when flipping a coin twice	

3. The times recorded for sprinters in a 100-meter race are considered a continuous random variable. Explain in one sentence why recording only whole-number seconds would be a problem in this context.

4. Scores on a standardized math test are normally distributed with a mean of 520 and a standard deviation of 90. Use the z-score formula to find the z-score for a student who scored 700.

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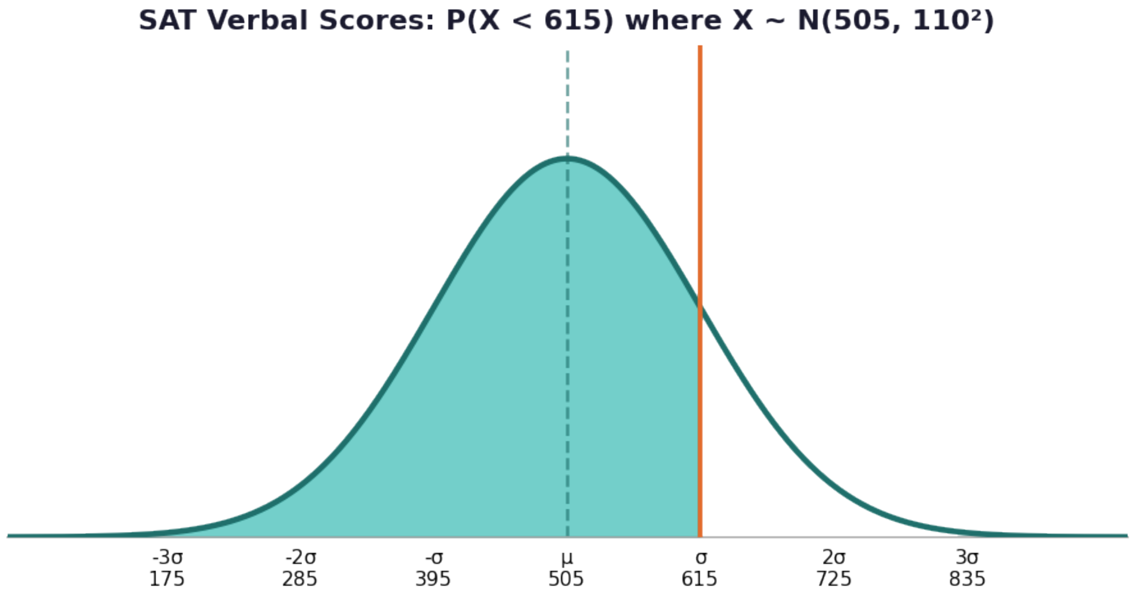
$$z = \frac{x - \mu}{\sigma}$$

5. Scores on the SAT verbal test follow an approximately normal distribution with a mean of 505 and a standard deviation of 110. Find the probability that a randomly selected student scored below 615. Shade the appropriate region on the normal curve.



6. Using the same SAT verbal distribution (mean = 505, standard deviation = 110), find the probability that a randomly selected student scored above 395.

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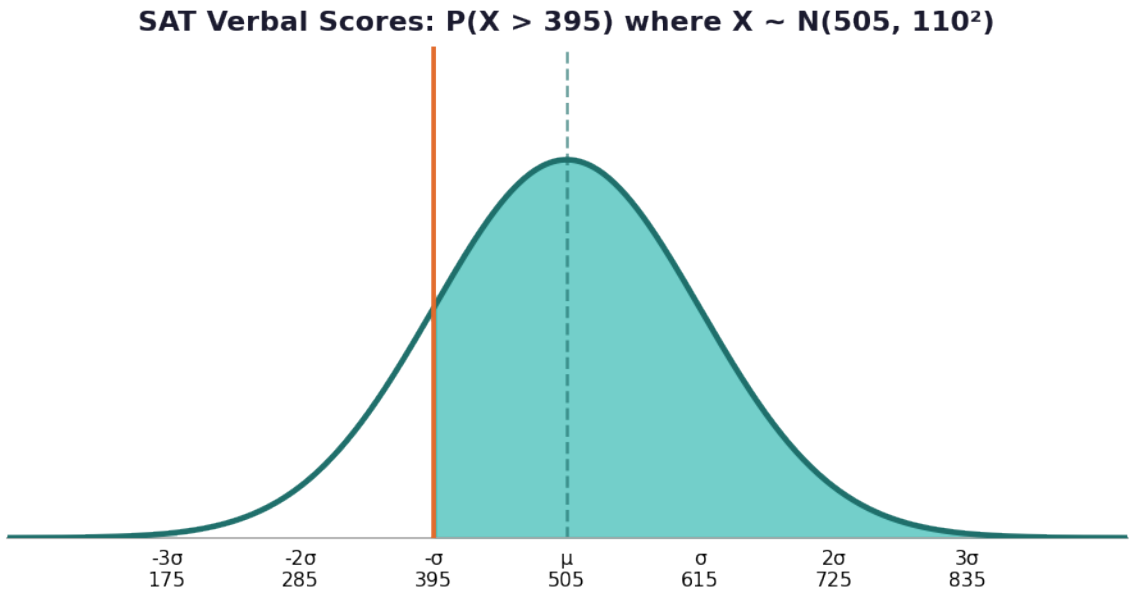
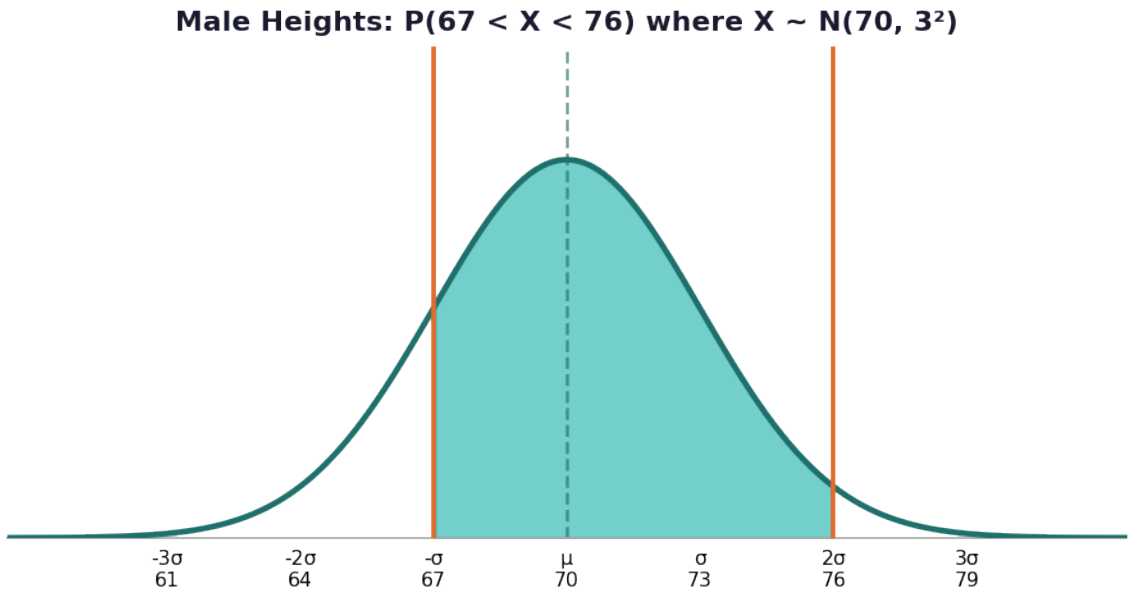


7. The heights of adult males are normally distributed with a mean of 70 inches and a standard deviation of 3 inches. Find the probability that a randomly selected male is between 67 and 76 inches tall.



8. SAT verbal scores are normally distributed with a mean of 505 and a standard deviation of 110. How high must a student score to be in the top 10% of all test takers? Use the z-score formula and solve for x.

$$z = \frac{x - \mu}{\sigma} \Rightarrow x = z\sigma + \mu$$

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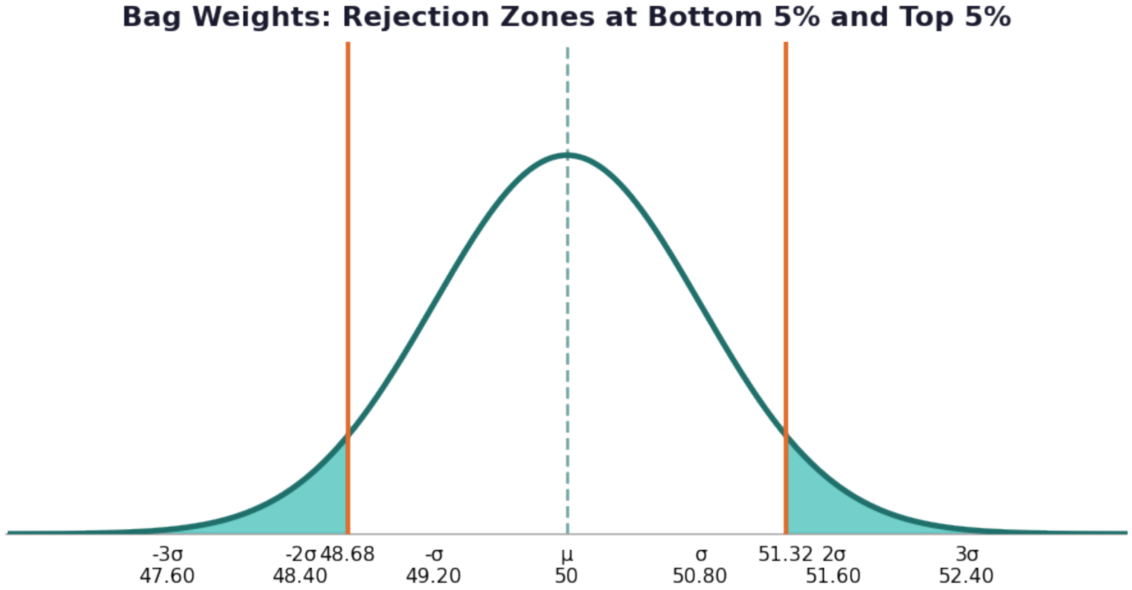
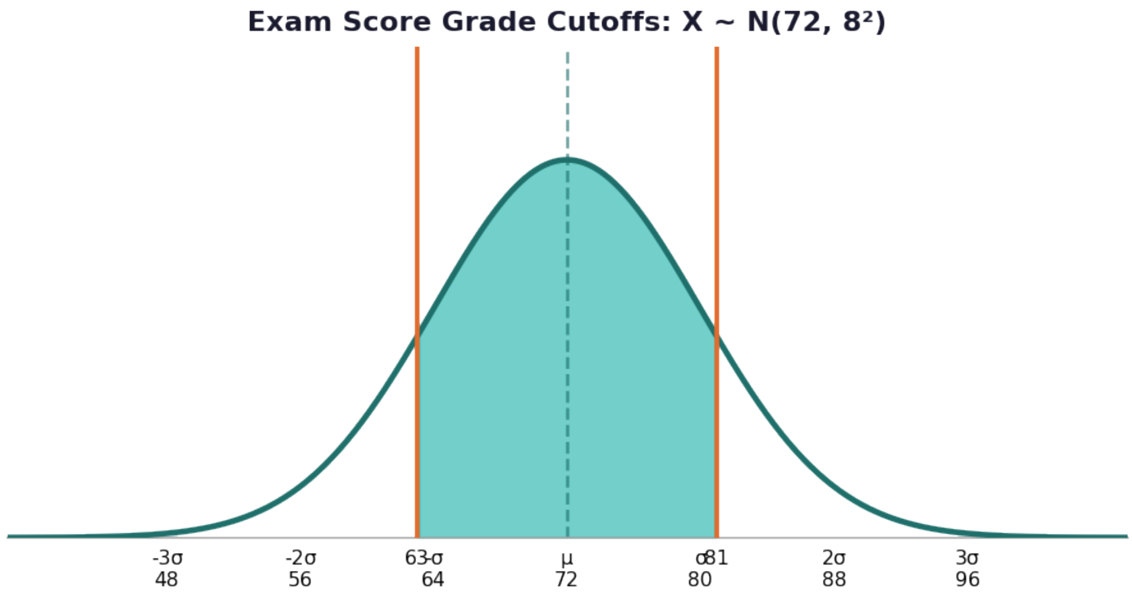
9. The weights of bags of flour produced by a factory are normally distributed with a mean of 50 lbs and a standard deviation of 0.8 lbs. Quality control rejects bags that fall in the bottom 5% or the top 5% of weights. Find the lower and upper cutoff weights.



10. At a university, final exam scores in a statistics course are normally distributed with a mean of 72 and a standard deviation of 8. Grades are assigned as follows: top 15% earn an A, the next 20% earn a B, the middle 30% earn a C, the next 20% earn a D, and the bottom 15% earn an F. Find the score cutoffs that separate each grade. Round to the nearest whole number.



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Discrete & Continuous Random Variables · Normal Distribution — Answer Key

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Answer Key

1. Answer: X = {0, 1, 2, 3, 4}; Discrete random variable

- A coin flipped 4 times can land on heads 0, 1, 2, 3, or 4 times.
- These outcomes are finite and countable, so X is a discrete random variable.
- Possible values: X = {0, 1, 2, 3, 4}

2. Answer: D, C, D, C, D

Scenario	D or C?
Number of students absent from school each day	D
The time it takes a runner to finish a 5K race	C
The number of cars in a parking lot	D
The weight of a newborn baby	C
The number of tails when flipping a coin twice	D

- Discrete: countable outcomes (absences, cars, tails from coin flips).
- Continuous: measured values that can be decimals (race time, baby weight).
- Results: D, C, D, C, D

3. Answer: Multiple runners would appear to have identical times (ties), making it impossible to rank them accurately.

- Without decimals, many runners would finish in the same whole-number second (e.g., both at '10 seconds').
- This creates false ties and loses the precision needed to distinguish performance differences.
- Therefore, decimal values must be recorded for accurate results.

4. Answer: z = 2.00

- Identify the values: $x = 700$, $\mu = 520$, $\sigma = 90$.
- Apply the formula: $z = (700 - 520) / 90$
- $z = 180 / 90 = 2.00$

5. Answer: P ≈ 0.8413 (84.13%)

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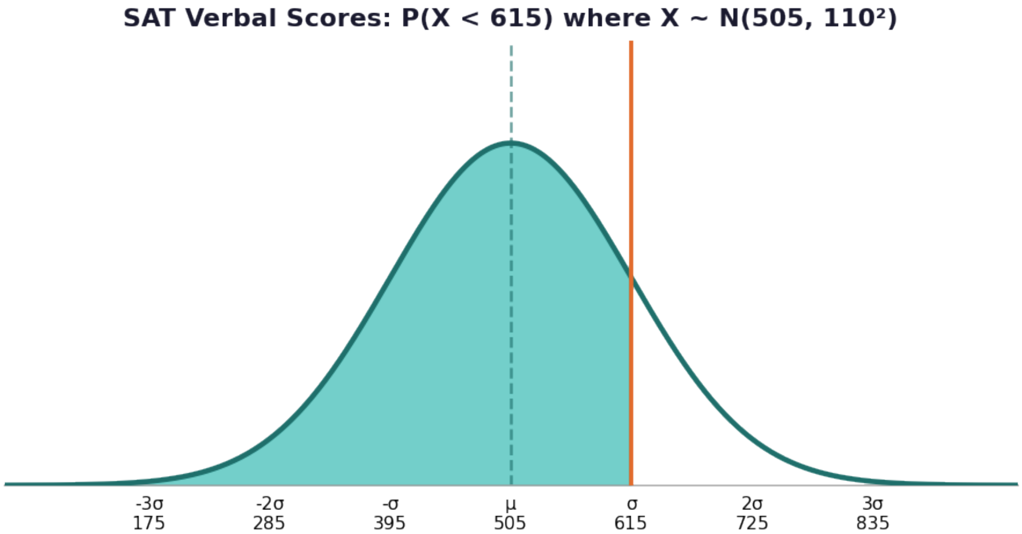
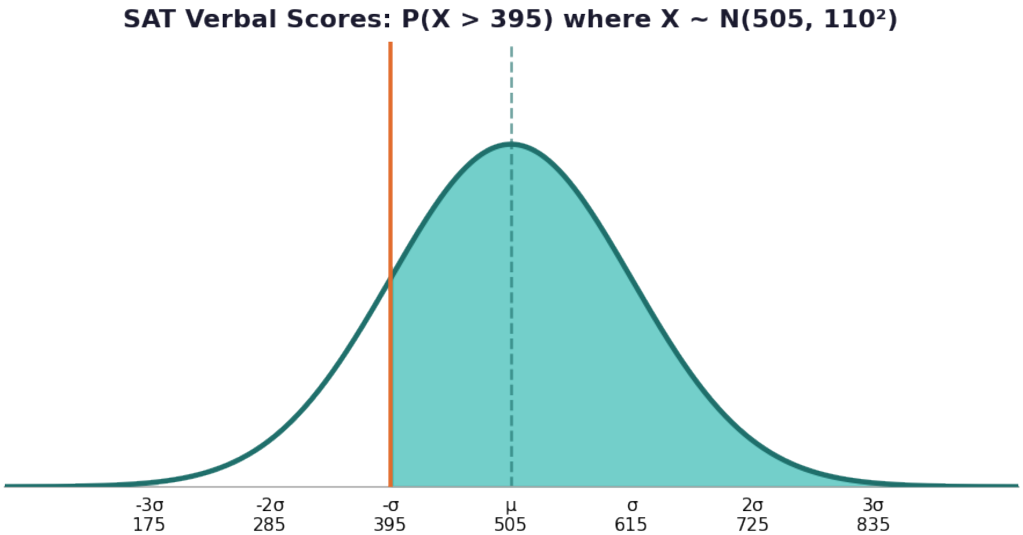


- Identify values: $x = 615$, $\mu = 505$, $\sigma = 110$.
- Calculate z: $z = (615 – 505) / 110 = 110 / 110 = 1.00$
- Look up $z = 1.00$ in the standard normal table.
- $P(X < 615) = P(Z < 1.00) \approx 0.8413$, or about 84.13%.

6. Answer: P ≈ 0.8413 (84.13%)



- Identify values: $x = 395$, $\mu = 505$, $\sigma = 110$.
- Calculate z: $z = (395 – 505) / 110 = –110 / 110 = –1.00$
- $P(X < 395) = P(Z < –1.00) \approx 0.1587$
- $P(X > 395) = 1 – 0.1587 = 0.8413$, or about 84.13%.

7. Answer: P ≈ 0.8186 (81.86%)

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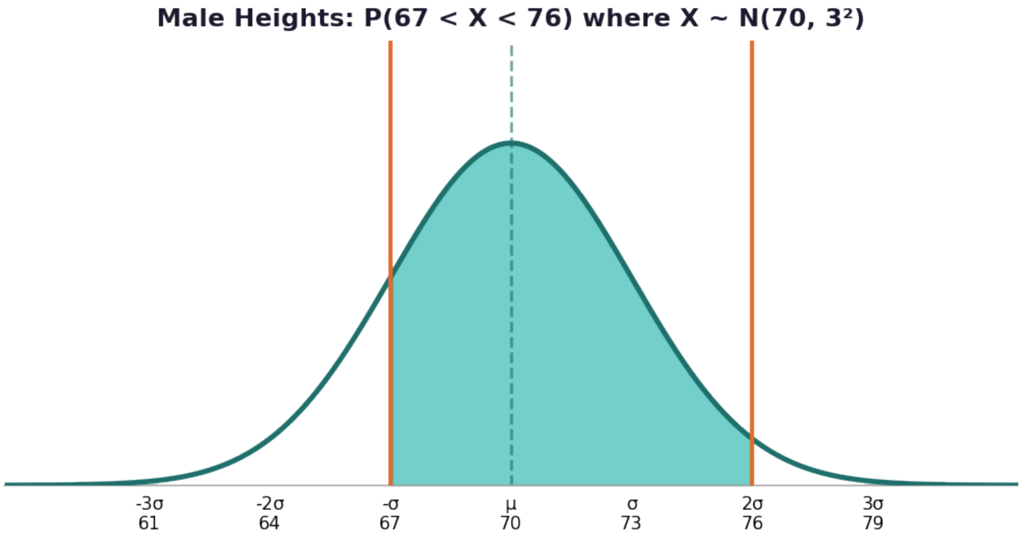

- For x■ = 67: z■ = (67 – 70) / 3 = –1.00 → P(Z < –1.00) ≈ 0.1587
- For x■ = 76: z■ = (76 – 70) / 3 = 2.00 → P(Z < 2.00) ≈ 0.9772
- P(67 < X < 76) = 0.9772 – 0.1587 = 0.8185 ≈ 81.86%

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8. Answer: x ≈ 645.8 (approximately 646 points)

- Top 10% means P(X > x) = 0.10, so P(X < x) = 0.90.
- Look up 0.90 in the z-table: z ≈ 1.28.
- Apply the inverse formula: x = zσ + μ = 1.28 × 110 + 505
- x = 140.8 + 505 = 645.8 ≈ 646 points.

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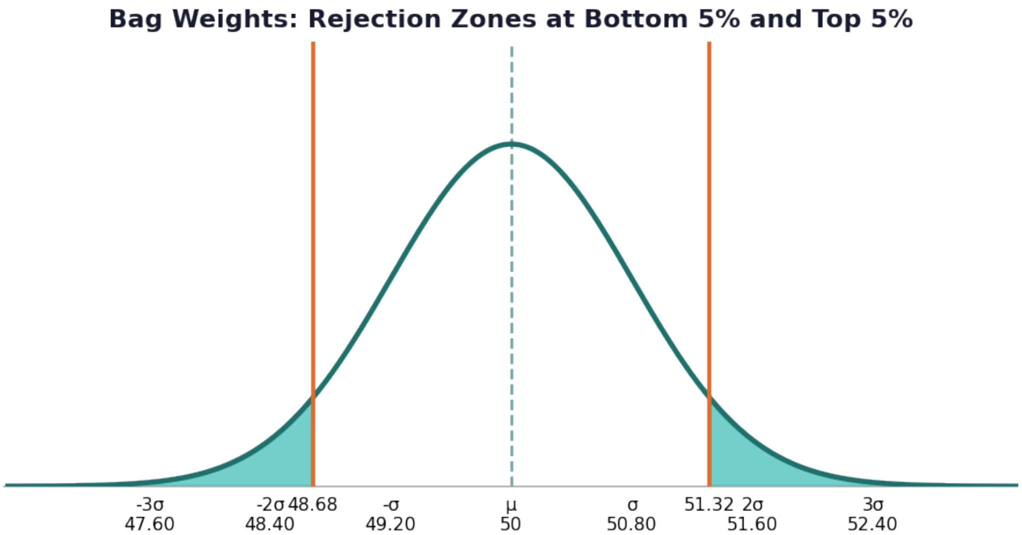
9. Answer: Lower cutoff ≈ 48.68 lbs; Upper cutoff ≈ 51.32 lbs



- Bottom 5%: P(X < x) = 0.05 → z ≈ –1.645
- Lower cutoff: x = –1.645 × 0.8 + 50 = –1.316 + 50 = 48.684 lbs
- Top 5%: P(X > x) = 0.05, so P(X < x) = 0.95 → z ≈ 1.645
- Upper cutoff: x = 1.645 × 0.8 + 50 = 1.316 + 50 = 51.316 lbs

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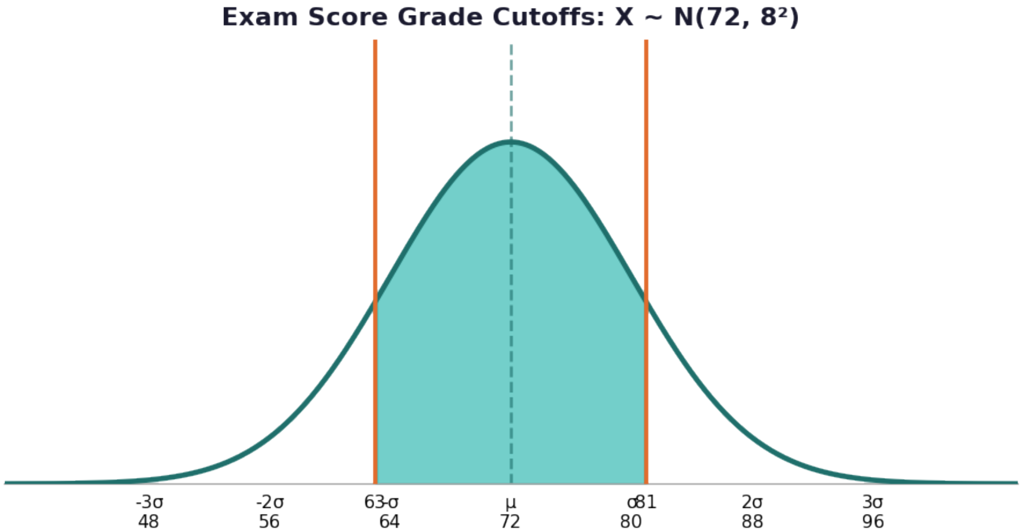
- Reject bags below approximately 48.68 lbs or above approximately 51.32 lbs.

10. Answer: F/D cutoff ≈ 64; D/C cutoff ≈ 66; C/B cutoff ≈ 78; B/A cutoff ≈ 80



- Bottom 15% (F/D cutoff): $P(X < x) = 0.15 \rightarrow z \approx -1.04 \rightarrow x = -1.04(8) + 72 = 63.68 \approx 64$
- Bottom 35% (D/C cutoff): $P(X < x) = 0.35 \rightarrow z \approx -0.39 \rightarrow x = -0.39(8) + 72 = 68.88 \approx 69$
- Bottom 65% (C/B cutoff): $P(X < x) = 0.65 \rightarrow z \approx 0.39 \rightarrow x = 0.39(8) + 72 = 75.12 \approx 75$
- Bottom 85% (B/A cutoff): $P(X < x) = 0.85 \rightarrow z \approx 1.04 \rightarrow x = 1.04(8) + 72 = 80.32 \approx 80$
- Grade cutoffs: F below 64, D from 64–68, C from 69–74, B from 75–79, A above 80.

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