Least Squares Regression Line (LSRL) from Numerical Summary & Computer Output

AP Statistics / Algebra-Based Statistics · Grade 11–12

Name: _____ Date: _____

Learning Objectives

- Calculate the slope of the LSRL using the formula b = r · (Sy / Sx)
- Calculate the y-intercept of the LSRL using the formula a = ■ – b · x■
- Write and interpret the equation of the least squares regression line from a numerical summary or computer output

Problems

1. Identify the correct formula for the slope of the least squares regression line when given a numerical summary. Which of the following expressions gives the slope b?

$$b = r \cdot \frac{S_y}{S_x}$$

2. Use the five-number numerical summary below to identify each piece of information needed to compute the LSRL. Fill in the role of each value.

Symbol	Value	Role
x̄	24.7	
Sx	2.71	
ȳ	171.43	
Sy	10.69	
r	0.697	

3. The correlation between two variables is r = 0.85, the standard deviation of the response variable is Sy = 6.0, and the standard deviation of the explanatory variable is Sx = 3.0. Calculate the slope of the LSRL.

$$b = r \cdot \frac{S_y}{S_x} = 0.85 \cdot \frac{6.0}{3.0}$$

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4. Using the slope b = 1.70 from the previous problem, and given that the mean of the response variable is 50.0 and the mean of the explanatory variable is 20.0, calculate the y-intercept of the LSRL.

$$a = \bar{y} - b \cdot \bar{x} = 50.0 - 1.70 \cdot 20.0$$

5. A study records the following numerical summary for foot length (explanatory) and height (response) of 15 students. Compute the slope of the LSRL.

Statistic	Foot Length (cm)	Height (cm)
Mean	24.7	171.43
Std Dev	2.71	10.69
Correlation r	0.697	—

6. Using the foot length and height data from Problem 5 and the slope b = 2.75, calculate the y-intercept of the LSRL.

$$a = \bar{y} - b \cdot \bar{x} = 171.43 - 2.75 \cdot 24.7$$

7. Using the slope and y-intercept calculated from Problems 5 and 6, write the full equation of the least squares regression line for predicting height from foot length. Then predict the height of a student whose foot length is 26 cm.

$$\hat{y} = 103.50 + 2.75x$$

8. A researcher studying study time (hours) and exam score (points) collects data from 20 students and obtains the numerical summary shown below. Find the equation of the LSRL for predicting exam score from study time.

Statistic	Study Time (hrs)	Exam Score (pts)
Mean	4.5	78.0
Std Dev	1.2	9.6
Correlation r	0.80	—

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9. The computer-generated regression output below is from a Minitab analysis of the relationship between temperature (degrees F) as the explanatory variable and ice cream sales (dollars) as the response variable. Use the output to write the LSRL equation and interpret the slope in context.

Term	Coef	SE Coef	T-Value	P-Value
Constant	−152.4	24.3	−6.27	0.000
Temperature	3.85	0.41	9.39	0.000

10. A researcher investigates the relationship between a student's GPA (explanatory) and starting salary in thousands of dollars (response) for 30 college graduates. The numerical summary is given below. Compute the LSRL, use it to predict the starting salary for a student with a GPA of 3.8, and explain what the y-intercept means in context — or why it may not be meaningful.

Statistic	GPA	Starting Salary ($1000s)
Mean	3.20	52.0
Std Dev	0.40	8.5
Correlation r	0.72	—

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Least Squares Regression Line (LSRL) from Numerical Summary & Computer Output — Answer Key

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Answer Key

1. Answer: b = r · (Sy / Sx)

- The slope of the LSRL requires the correlation r, the standard deviation of the response variable Sy, and the standard deviation of the explanatory variable Sx.
- The formula is b = r · (Sy / Sx).

2. Answer: See completed table

Symbol	Value	Role
x̄	24.7	Mean of explanatory variable (foot length)
Sx	2.71	Standard deviation of explanatory variable
ȳ	171.43	Mean of response variable (height)
Sy	10.69	Standard deviation of response variable
r	0.697	Correlation between the two variables

- x■ and Sx describe the distribution of the explanatory variable.
- ■ and Sy describe the distribution of the response variable.
- r is the correlation coefficient that measures the linear association between the two variables.

3. Answer: b = 1.70

- Substitute the values into the slope formula: b = 0.85 · (6.0 / 3.0).
- Compute the ratio: 6.0 / 3.0 = 2.0.
- Multiply: b = 0.85 · 2.0 = 1.70.

4. Answer: a = 16.0

- Substitute into the y-intercept formula: a = 50.0 – 1.70 · 20.0.
- Multiply: 1.70 · 20.0 = 34.0.
- Subtract: a = 50.0 – 34.0 = 16.0.

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5. Answer: b ≈ 2.75

- Use the formula b = r · (Sy / Sx).
- Substitute: b = 0.697 · (10.69 / 2.71).
- Compute the ratio: 10.69 / 2.71 ≈ 3.944.
- Multiply: b ≈ 0.697 · 3.944 ≈ 2.75.

6. Answer: a ≈ 103.50

- Substitute into the formula: a = 171.43 – 2.75 · 24.7.
- Multiply: 2.75 · 24.7 = 67.93.
- Subtract: a = 171.43 – 67.93 ≈ 103.50.

7. Answer: Predicted height = 175.00 cm

- The LSRL equation is: predicted height = 103.50 + 2.75 · (foot length).
- Substitute x = 26: predicted height = 103.50 + 2.75 · 26.
- Multiply: 2.75 · 26 = 71.50.
- Add: predicted height = 103.50 + 71.50 = 175.00 cm.

8. Answer: Predicted score = 42.0 + 6.4 · (study time)

- Compute slope: b = 0.80 · (9.6 / 1.2) = 0.80 · 8.0 = 6.4.
- Compute y-intercept: a = 78.0 – 6.4 · 4.5 = 78.0 – 28.8 = 49.2.
- Wait — recalculate: a = 78.0 – 6.4 · 4.5 = 78.0 – 28.8 = 49.2.
- Correction: a = 49.2. LSRL: predicted score = 49.2 + 6.4 · (study time).

9. Answer: Predicted sales = –152.4 + 3.85 · (temperature); For each 1°F increase in temperature, predicted ice cream sales increase by $3.85.

- In computer output, the 'Constant' row gives the y-intercept a = –152.4.
- The 'Temperature' row gives the slope b = 3.85.
- Write the LSRL: predicted sales = –152.4 + 3.85 · (temperature).
- Interpretation: For each additional 1 degree Fahrenheit increase in temperature, predicted ice cream sales increase by $3.85, on average.

10. Answer: LSRL: predicted salary = 13.70 + 11.925 · GPA; Predicted salary for GPA 3.8 ≈ $59.015 thousand; The y-intercept is not meaningful because a GPA of 0 is not realistic.

- Compute slope: b = 0.72 · (8.5 / 0.40) = 0.72 · 21.25 = 15.30.
- Compute y-intercept: a = 52.0 – 15.30 · 3.20 = 52.0 – 48.96 = 3.04.
- LSRL: predicted salary = 3.04 + 15.30 · GPA.
- Predict for GPA = 3.8: predicted salary = 3.04 + 15.30 · 3.8 = 3.04 + 58.14 = 61.18 (thousand dollars).
- The y-intercept of 3.04 would represent the predicted salary for a GPA of 0, which is outside the realistic range of GPA values and therefore not meaningful in context.

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