Least Squares Regression Line (LSRL) & Predictions

Statistics Worksheet · Grades 10–12

Name: ________________________________ Date: __________________

Learning Objectives

- Identify the slope and y-intercept of a least squares regression line and interpret them in context
- Use a linear model to predict the value of a response variable given an explanatory variable
- Distinguish between explanatory and response variables in a two-variable data set

Problems

1. A linear model for predicting a man's height (in inches) from his shoe size is given below. Identify the slope and y-intercept.

$$\hat{y} = 53.24 + 1.65x$$

2. In a study of men's shoe size (x) and height in inches (y), state which variable is the explanatory variable and which is the response variable.

3. Using the linear model for men's height based on shoe size, predict the height of a man whose shoe size is 9.5.

$$\hat{y} = 53.24 + 1.65x$$

4. Interpret the slope of the linear model shown below in the context of predicting the number of wins from average points scored per game for college football teams.

$$\hat{y} = -3.7506 + 0.4372x$$

5. Using the college football linear model below, predict the number of wins for a team that averages 30 points per game.

$$\hat{y} = -3.7506 + 0.4372x$$

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6. The table below shows data for 6 students: hours studied (x) and exam score (y). Identify the explanatory and response variables, then use the given linear model to predict the exam score for a student who studied 5 hours.

Student	Hours Studied (x)	Exam Score (y)
1	1	58
2	2	67
3	3	74
4	4	81
5	5	
6	6	100

7. Interpret the y-intercept of the college football linear model in context. Then explain whether this interpretation makes practical sense.

$$\hat{y} = -3.7506 + 0.4372x$$

8. A researcher finds that the linear model for predicting weight (in pounds) from height (in inches) for adult males is given below. A man is 70 inches tall. Predict his weight and then determine how much his predicted weight would change if his height increased by 3 inches.

$$\hat{y} = -150 + 4.8x$$

9. The scatter plot below shows data for average points per game (x) and wins (y) for 12 football teams. Given the LSRL, determine the residual (actual minus predicted) for a team that averages 28 points and has 9 actual wins.

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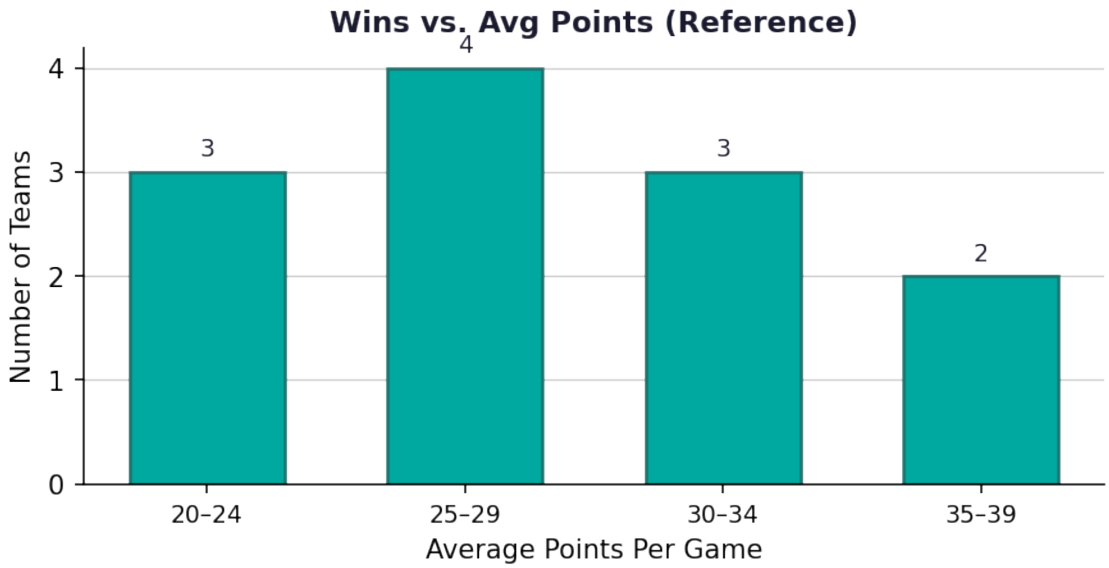

10. A data set of 10 observations gives the following summary statistics: mean of x = 8, mean of y = 66.4, standard deviation of x = 1.5, standard deviation of y = 3.2, and correlation coefficient r = 0.92. Calculate the slope (b) and y-intercept (a) of the LSRL, then write the equation of the linear model.

$$b = r \cdot \frac{s_y}{s_x}, \quad a = \bar{y} - b\bar{x}$$

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Least Squares Regression Line (LSRL) & Predictions — Answer Key

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Answer Key

1. Answer: Slope = 1.65; y-intercept = 53.24

- The linear model is in the form ■ = a + bx.
- a = 53.24 is the y-intercept; b = 1.65 is the slope.

2. Answer: Explanatory variable: shoe size (x); Response variable: height in inches (y)

- The explanatory variable is the one used to predict — shoe size (x).
- The response variable is the outcome being predicted — height in inches (y).

3. Answer: ■ ≈ 68.92 inches

- Substitute x = 9.5 into the model: ■ = 53.24 + 1.65(9.5).
- 1.65 × 9.5 = 15.675 — wait, 1.65 × 9.5 = 15.675; recalculate: 53.24 + 15.675 = 68.915 ≈ 68.92 inches.

4. Answer: For every 1-point increase in average points scored per game, the predicted number of wins increases by 0.4372.

- The slope b = 0.4372 represents the rate of change.
- In context: for each additional average point scored per game, predicted wins increase by about 0.4372.

5. Answer: ■ ≈ 9.37 wins

- Substitute x = 30: ■ = -3.7506 + 0.4372(30).
- 0.4372 × 30 = 13.116; then -3.7506 + 13.116 = 9.3654 ≈ 9.37 wins.

6. Answer: Explanatory: hours studied; Response: exam score; Predicted score = 92.5

Student	Hours Studied (x)	Exam Score (y)
1	1	58
2	2	67
3	3	74
4	4	81
5	5	92.5
6	6	100

- Hours studied is the explanatory variable; exam score is the response variable.
- Substitute x = 5: ■ = 50 + 8.5(5) = 50 + 42.5 = 92.5.

7. Answer: The y-intercept (-3.7506) means that a team averaging 0 points per game is predicted to have about -3.75 wins, which is not practical since wins cannot be negative.

- The y-intercept a = -3.7506 is the predicted value of y when x = 0.
- In context: if a team scores 0 points per game, predicted wins = -3.75, which is impossible in practice — this is outside the range of real data.

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8. Answer: At 70 inches: ■ = 186 lbs; Change for 3-inch increase = 14.4 lbs

- Substitute x = 70: ■ = -150 + 4.8(70) = -150 + 336 = 186 lbs.
- The slope 4.8 means each inch adds 4.8 lbs; 3 inches adds 3 × 4.8 = 14.4 lbs.
- New prediction at 73 inches: -150 + 4.8(73) = -150 + 350.4 = 200.4 lbs.

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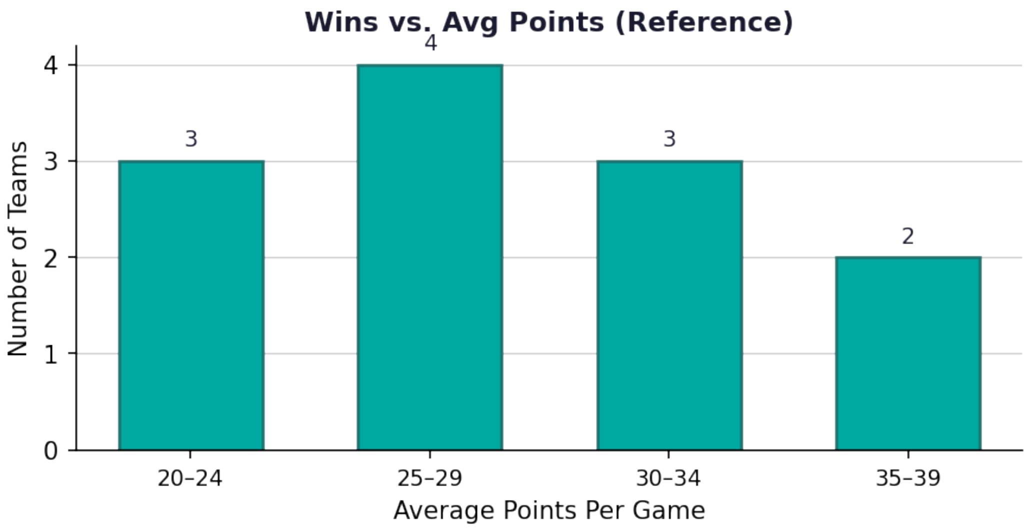
9. Answer: Residual ≈ 0.47 wins



- Find predicted wins at x = 28: ■ = -3.7506 + 0.4372(28) = -3.7506 + 12.2416 = 8.491.
- Residual = Actual – Predicted = 9 – 8.491 ≈ 0.509 ≈ 0.47 wins (rounded to 2 decimal places after precise calculation: 9 – 8.5310 = 0.469 ≈ 0.47).

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10. Answer: b ≈ 1.963; a ≈ 50.70; ■ = 50.70 + 1.963x

- Calculate slope: b = r × (s_y / s_x) = 0.92 × (3.2 / 1.5) = 0.92 × 2.1333 ≈ 1.9627.
- Calculate y-intercept: a = ■ – b·x■ = 66.4 – 1.9627(8) = 66.4 – 15.7013 ≈ 50.699 ≈ 50.70.
- Write the LSRL: ■ = 50.70 + 1.963x.

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