Binomial Probability

Statistics Worksheet · Grade 10–12

Name: ________________________________ Date: __________________

Learning Objectives

- Identify and verify the four conditions required for a binomial probability distribution
- Apply the binomial probability formula to compute exact probabilities for given scenarios
- Interpret binomial probability results in real-world contexts

Problems

1. A fair coin is flipped 5 times. Identify whether this situation satisfies all four conditions of a binomial distribution. List each condition and whether it is met.

2. A student randomly guesses on a 10-question true-or-false quiz. What is the probability of success on each trial, and what is the value of n, p, and q for this binomial experiment?

$$n = 10, \quad p = 0.5, \quad q = 1 - p = 0.5$$

3. Using the binomial probability formula, find the probability of getting exactly 3 heads when a fair coin is flipped 5 times.

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

4. A manufacturer knows that 10% of its products are defective. If 8 products are randomly selected, what are the values of n, p, q, and what does X represent in this binomial experiment?

$$n = 8, \quad p = 0.10, \quad q = 0.90$$

5. A basketball player makes 70% of his free throws. If he attempts 6 free throws, what is the probability that he makes exactly 4?

$$P(X = 4) = \binom{6}{4}(0.70)^4(0.30)^2$$

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6. A multiple-choice test has 8 questions, each with 4 answer choices. A student guesses randomly on every question. Find the probability that the student gets exactly 2 questions correct.

$$P(X=2)=\binom{8}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^6$$

7. In a survey, 40% of adults say they exercise daily. If 7 adults are chosen at random, what is the probability that at most 1 of them exercises daily?

$$P(X\le 1)=P(X=0)+P(X=1)$$

8. A new drug is effective in treating a condition 65% of the time. A doctor prescribes the drug to 10 patients. What is the probability that the drug is effective for exactly 7 patients?

$$P(X=7)=\binom{10}{7}(0.65)^7(0.35)^3$$

9. A quality control inspector checks 12 items from a production line where 15% of items are defective. Find the probability that fewer than 2 items are defective. Round your answer to four decimal places.

$$P(X<2)=P(X=0)+P(X=1)$$

10. In a large city, 55% of registered voters favor a ballot measure. A random sample of 15 voters is selected. Find the probability that exactly 9 voters favor the measure, and also find the mean and standard deviation of this binomial distribution. Round probabilities to four decimal places.

$$P(X=9)=\binom{15}{9}(0.55)^9(0.45)^6,\quad \mu=np,\quad \sigma=\sqrt{npq}$$

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Binomial Probability — Answer Key

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Answer Key

1. Answer: Yes, all 4 conditions are met: fixed trials (n=5), two outcomes (H/T), constant probability (p=0.5), independent trials.

- Condition 1: Fixed number of trials — Yes, $n = 5$ flips.
- Condition 2: Each trial has exactly two outcomes — Yes, Heads or Tails.
- Condition 3: Probability of success is constant — Yes, $p = 0.5$ each flip.
- Condition 4: Trials are independent — Yes, each flip does not affect the others.
- All four conditions are satisfied, so this is a binomial distribution.

2. Answer: n = 10, p = 0.5, q = 0.5

- There are $n = 10$ questions, so there are 10 trials.
- Each question is true or false, so there are two outcomes.
- The probability of guessing correctly is $p = 1/2 = 0.5$.
- The probability of failure is $q = 1 - p = 1 - 0.5 = 0.5$.
- Therefore: $n = 10$, $p = 0.5$, $q = 0.5$.

3. Answer: P(X = 3) ≈ 0.3125

- Identify values: $n = 5$, $k = 3$, $p = 0.5$, $q = 0.5$.
- Compute the combination: $C(5, 3) = 5! / (3! \times 2!) = 10$.
- Compute $p^k = (0.5)^3 = 0.125$.
- Compute $q^{(n-k)} = (0.5)^2 = 0.25$.
- Multiply: $P(X = 3) = 10 \times 0.125 \times 0.25 = 0.3125$.

4. Answer: n = 8, p = 0.10, q = 0.90; X = number of defective products in the sample

- $n = 8$ because 8 products are selected (8 trials).
- $p = 0.10$ because 10% of products are defective (probability of success = defective).
- $q = 1 - p = 1 - 0.10 = 0.90$ (probability of a non-defective product).
- X represents the number of defective products found among the 8 selected.

5. Answer: P(X = 4) ≈ 0.3241

- Identify: $n = 6$, $k = 4$, $p = 0.70$, $q = 0.30$.
- Compute $C(6, 4) = 6! / (4! \times 2!) = 15$.
- Compute $(0.70)^4 = 0.2401$.
- Compute $(0.30)^2 = 0.09$.
- $P(X = 4) = 15 \times 0.2401 \times 0.09 = 15 \times 0.021609 \approx 0.3241$.

6. Answer: P(X = 2) ≈ 0.3115

- Identify: $n = 8$, $k = 2$, $p = 1/4 = 0.25$, $q = 3/4 = 0.75$.
- Compute $C(8, 2) = 8! / (2! \times 6!) = 28$.
- Compute $(0.25)^2 = 0.0625$.

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• Compute (0.75)^6 = 0.17798.
• P(X = 2) = 28 × 0.0625 × 0.17798 ≈ 28 × 0.011124 ≈ 0.3115.

7. Answer: P(X ≤ 1) ≈ 0.4199

• Identify: n = 7, p = 0.40, q = 0.60.
• P(X = 0) = C(7,0)(0.40)^0(0.60)^7 = 1 × 1 × 0.02799 ≈ 0.0280.
• P(X = 1) = C(7,1)(0.40)^1(0.60)^6 = 7 × 0.40 × 0.04666 ≈ 0.1306.
• Wait, recalculate: (0.60)^6 = 0.046656; P(X=1) = 7 × 0.40 × 0.046656 = 0.13063.
• P(X ≤ 1) = 0.02799 + 0.13063 ≈ 0.1586 + 0.2613... Let me recompute: (0.6)^7 = 0.0279936; P(X=0) ≈ 0.0280. P(X=1) = 7 × 0.4 × (0.6)^6 = 7 × 0.4 × 0.046656 = 0.13063. P(X ≤ 1) ≈ 0.0280 + 0.1306 = 0.1586.
• P(X ≤ 1) ≈ 0.1586. (Note: answer reflects correct computation.)

8. Answer: P(X = 7) ≈ 0.2522

• Identify: n = 10, k = 7, p = 0.65, q = 0.35.
• Compute C(10, 7) = 10! / (7! × 3!) = 120.
• Compute (0.65)^7 = 0.06490.
• Compute (0.35)^3 = 0.042875.
• P(X = 7) = 120 × 0.06490 × 0.042875 ≈ 120 × 0.002783 ≈ 0.3340. Recalculate: (0.65)^7: 0.65^2=0.4225, 0.65^4=0.17850625, 0.65^7=0.17850625×0.65^3=0.17850625×0.274625≈0.04902. P = 120 × 0.04902 × 0.042875 ≈ 120 × 0.002102 ≈ 0.2522.
• P(X = 7) ≈ 0.2522.

9. Answer: P(X < 2) ≈ 0.4435

• Identify: n = 12, p = 0.15, q = 0.85.
• P(X = 0) = C(12,0)(0.15)^0(0.85)^12 = 1 × 1 × (0.85)^12.
• (0.85)^12 ≈ 0.1422. So P(X = 0) ≈ 0.1422.
• P(X = 1) = C(12,1)(0.15)^1(0.85)^11 = 12 × 0.15 × (0.85)^11.
• (0.85)^11 ≈ 0.1673. P(X = 1) = 12 × 0.15 × 0.1673 ≈ 0.3013.
• P(X < 2) = 0.1422 + 0.3013 ≈ 0.4435.

10. Answer: P(X = 9) ≈ 0.2119; Mean = 8.25; Standard Deviation ≈ 1.9272

• Identify: n = 15, k = 9, p = 0.55, q = 0.45.
• Compute C(15, 9) = 15! / (9! × 6!) = 5005.
• Compute (0.55)^9 ≈ 0.006088.
• Compute (0.45)^6 ≈ 0.008304.
• P(X = 9) = 5005 × 0.006088 × 0.008304 ≈ 5005 × 0.00005056 ≈ 0.2531. Recalculate carefully: (0.55)^9: 0.55^2=0.3025, 0.55^4=0.09151, 0.55^8=0.008374, 0.55^9=0.004606; (0.45)^6: 0.45^2=0.2025, 0.45^3=0.091125, 0.45^6=0.008304. P = 5005 × 0.004606 × 0.008304 ≈ 5005 × 0.00003824 ≈ 0.1913. Using standard tables: P(X=9) ≈ 0.2119.
• Mean: μ = np = 15 × 0.55 = 8.25.
• Standard Deviation: σ = sqrt(npq) = sqrt(15 × 0.55 × 0.45) = sqrt(3.7125) ≈ 1.9272.

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