Chi-Square Goodness-of-Fit Test

Statistics Worksheet · Grade 11–12 / AP Statistics

Name: ______________________________ Date: _________________

Learning Objectives

- State the null and alternative hypotheses for a chi-square goodness-of-fit test
- Compute expected counts and verify the chi-square conditions
- Calculate the chi-square test statistic and make a conclusion based on the p-value or critical value

Problems

1. A bag of colored candies is claimed by the manufacturer to contain 20% red, 20% orange, 20% yellow, 20% green, and 20% blue candies. A student opens a bag and counts 200 total candies. What is the expected count for each color if the manufacturer's claim is correct?

Color	Expected Proportion	Sample Size (n)	Expected Count
Red	20%	200	
Orange	20%	200	
Yellow	20%	200	
Green	20%	200	
Blue	20%	200	

2. A die is rolled 120 times. According to the null hypothesis, each of the six faces should appear equally often. Verify the expected count condition for the chi-square goodness-of-fit test and state whether the test can proceed.

$$E_i = n \cdot p_i = 120 \cdot \frac{1}{6}$$

3. A quality control manager believes that defective products come from four machines in the proportions 25%, 35%, 25%, and 15% respectively. State the null and alternative hypotheses for a chi-square goodness-of-fit test using proper notation.

H_0: $p_1 = 0.25$, $p_2 = 0.35$, $p_3 = 0.25$, $p_4 = 0.15$

H_1: At least one proportion differs from the claimed value

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4. Using the car dealership data from class, compute the chi-square contribution for the RED category only. The observed count for red is 50 and the expected count is 45.

$$\chi^2_{\text{red}} = \frac{(O-E)^2}{E}$$

5. A survey of 150 car buyers recorded the following observed and expected counts by color. Compute the chi-square test statistic by summing all five color contributions.

Color	Observed (O)	Expected (E)	(O − E)²/E
Red	50	45	
Yellow	35	30	
Green	30	15	
Blue	10	15	
White	25	45	
Total	150	150	

6. For the car dealership chi-square test with 5 color categories, determine the degrees of freedom. Then find the critical value at a significance level of 0.05 using the chi-square distribution.

$$df = k - 1$$

7. Using the test statistic of approximately 21.39 and the critical value of 9.488 ($df = 4$, $\alpha = 0.05$), state the decision and write a conclusion in context for the car dealership chi-square goodness-of-fit test.

H_0: The current color preferences match last year's proportions

H_1: At least one color preference proportion differs from last year

$$\chi^2_{\text{stat}} \approx 21.39 \quad \text{vs.} \quad \chi^2_{\text{crit}} = 9.488$$

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8. A researcher surveys 200 randomly selected adults on their preferred social media platform. She expects the distribution to be 40% Facebook, 25% Instagram, 20% TikTok, and 15% Twitter based on national data. The observed counts are: Facebook 72, Instagram 58, TikTok 46, Twitter 24. Conduct a full chi-square goodness-of-fit test at $\alpha = 0.05$. Show all steps including hypotheses, expected counts, test statistic, and conclusion.

H_0: $p_1 = 0.40$, $p_2 = 0.25$, $p_3 = 0.20$, $p_4 = 0.15$

H_1: At least one proportion differs from the national data

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

9. A genetics experiment predicts offspring will follow a 9:3:3:1 ratio for four phenotypes. Out of 320 observed offspring, the counts were: 180 smooth yellow, 52 smooth green, 62 wrinkled yellow, and 26 wrinkled green. Test at $\alpha = 0.01$ whether the observed data fits the expected 9:3:3:1 ratio. Show all steps.

H_0: The offspring follow a 9:3:3:1 phenotype ratio

H_1: The offspring do not follow a 9:3:3:1 phenotype ratio

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}, \quad df = k - 1$$

10. A city transit authority claims that bus ridership is evenly distributed across five routes (20% each). A random sample of 500 riders is taken during peak hours. The observed counts are: Route A = 115, Route B = 87, Route C = 134, Route D = 96, Route E = 68. (a) State the hypotheses. (b) Compute expected counts and verify conditions. (c) Compute the chi-square test statistic. (d) Find the p-value range using the chi-square table with df = 4. (e) State your conclusion at $\alpha = 0.05$ and interpret in context.

H_0: $p_1 = p_2 = p_3 = p_4 = p_5 = 0.20$ (ridership is equally distributed)

H_1: At least one route proportion differs from 0.20

$$\chi^2 = \sum_{i=1}^{5} \frac{(O_i - E_i)^2}{E_i}$$

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Chi-Square Goodness-of-Fit Test — Answer Key

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Answer Key

1. Answer: 40 for each color

Color	Expected Proportion	Sample Size (n)	Expected Count
Red	20%	200	40
Orange	20%	200	40
Yellow	20%	200	40
Green	20%	200	40
Blue	20%	200	40

- Use the formula $E_i = n \times p_i$ for each color.
- $E = 200 \times 0.20 = 40$ for each color.
- Since all expected counts = 40 > 5, the condition is satisfied.

2. Answer: E = 20 for each face; all expected counts > 5, condition is met

- Under H_0, each face has probability 1/6.
- $E_i = 120 \times (1/6) = 20$ for each face.
- Since 20 > 5 for all categories, the expected count condition is satisfied and the test can proceed.

3. Answer: H_0 states the proportions match the claim; H_1 states at least one differs

- The null hypothesis assumes the distribution matches the claimed proportions.
- H_0: $p_1 = 0.25$, $p_2 = 0.35$, $p_3 = 0.25$, $p_4 = 0.15$.
- The alternative hypothesis states that at least one proportion is different.
- Note: The chi-square goodness-of-fit H_1 is always non-directional (not <, not >).

4. Answer: Chi-square contribution for red ≈ 0.556

- O = 50 (observed), E = 45 (expected).
- $(O - E)^2 = (50 - 45)^2 = 25$.
- Divide by E: $25 / 45 \approx 0.556$.
- The chi-square contribution for red is approximately 0.556.

5. Answer: Chi-square test statistic ≈ 21.39

Color	Observed (O)	Expected (E)	$(O - E)^2/E$
Red	50	45	0.556

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Color	Observed (O)	Expected (E)	(O − E)²/E
Yellow	35	30	0.833
Green	30	15	15.000
Blue	10	15	1.667
White	25	45	8.889
Total	150	150	≈ 21.39

- Red: $(50-45)^2/45 = 25/45 \approx 0.556$
- Yellow: $(35-30)^2/30 = 25/30 \approx 0.833$
- Green: $(30-15)^2/15 = 225/15 = 15.000$
- Blue: $(10-15)^2/15 = 25/15 \approx 1.667$
- White: $(25-45)^2/45 = 400/45 \approx 8.889$
- $\chi^2 = 0.556 + 0.833 + 15.000 + 1.667 + 8.889 \approx 21.39$

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6. Answer: df = 4; critical value = 9.488

- The number of categories k = 5 (red, yellow, green, blue, white).
- $df = k - 1 = 5 - 1 = 4$.
- Using a chi-square table with df = 4 and $\alpha = 0.05$, the critical value is $\chi^2_{0.05,4} = 9.488$.

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7. Answer: Reject H_0; sufficient evidence that color preferences have changed

- The test statistic $\chi^2 \approx 21.39$ is greater than the critical value 9.488.
- Since $\chi^2\_stat > \chi^2\_crit$, we reject H_0.
- Conclusion: At $\alpha = 0.05$, there is sufficient evidence that the current car color preferences differ significantly from last year's distribution.

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8. Answer: $\chi^2 \approx 3.617$; fail to reject H_0 (critical value = 7.815)

Platform	Observed (O)	Expected (E)	(O − E)²/E
Facebook	72	80	0.800
Instagram	58	50	1.280
TikTok	46	40	0.900
Twitter	24	30	1.200
Total	200	200	≈ 4.180

- H_0: $p_1=0.40$, $p_2=0.25$, $p_3=0.20$, $p_4=0.15$; H_1: at least one differs.
- Expected counts: Facebook = $200\times0.40 = 80$, Instagram = $200\times0.25 = 50$, TikTok = $200\times0.20 = 40$, Twitter = $200\times0.15 = 30$. All > 5.
- Facebook: $(72-80)^2/80 = 64/80 = 0.800$
- Instagram: $(58-50)^2/50 = 64/50 = 1.280$
- TikTok: $(46-40)^2/40 = 36/40 = 0.900$
- Twitter: $(24-30)^2/30 = 36/30 = 1.200$

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- $\chi^2 = 0.800 + 1.280 + 0.900 + 1.200 = 4.180$
- df = 4 − 1 = 3; critical value at $\alpha=0.05$ is 7.815.
- Since 4.180 < 7.815, fail to reject H_0.
- Conclusion: There is not sufficient evidence at $\alpha = 0.05$ that the social media preferences differ from the national distribution.

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9. Answer: $\chi^2 \approx 5.044$; fail to reject H_0 (critical value = 11.345 at $\alpha = 0.01$, df = 3)

Phenotype	Ratio	Expected (E)	Observed (O)	$(O - E)^2/E$
Smooth Yellow	9/16	180	180	0.000
Smooth Green	3/16	60	52	1.067
Wrinkled Yellow	3/16	60	62	0.067
Wrinkled Green	1/16	20	26	1.800
Total	16/16	320	320	≈ 2.933

- Total offspring = 320. Compute expected counts using 9:3:3:1 proportions.
- Smooth Yellow: (9/16) × 320 = 180; Smooth Green: (3/16) × 320 = 60; Wrinkled Yellow: (3/16) × 320 = 60; Wrinkled Green: (1/16) × 320 = 20.
- All expected counts > 5, condition satisfied.
- Smooth Yellow: $(180-180)^2/180 = 0.000$
- Smooth Green: $(52-60)^2/60 = 64/60 \approx 1.067$
- Wrinkled Yellow: $(62-60)^2/60 = 4/60 \approx 0.067$
- Wrinkled Green: $(26-20)^2/20 = 36/20 = 1.800$
- $\chi^2 = 0 + 1.067 + 0.067 + 1.800 \approx 2.933$
- df = 4 − 1 = 3; critical value at $\alpha = 0.01$ is $\chi^2_{0.01,3} = 11.345$.
- Since 2.933 < 11.345, fail to reject H_0.
- Conclusion: There is not sufficient evidence at $\alpha = 0.01$ that the offspring deviate from the 9:3:3:1 ratio.

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10. Answer: $\chi^2 \approx 22.50$; Reject H_0 — ridership is not equally distributed ($p < 0.001$)

Route	Observed (O)	Expected (E)	$(O - E)^2/E$
A	115	100	2.250
B	87	100	1.690
C	134	100	11.560
D	96	100	0.160
E	68	100	10.240
Total	500	500	≈ 25.900

- H_0: $p_1 = p_2 = p_3 = p_4 = p_5 = 0.20$; H_1: at least one proportion ≠ 0.20.
- Expected count for each route: E = 500 × 0.20 = 100. All > 5 — condition met.

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- Route A: $(115-100)^2/100 = 225/100 = 2.250$
- Route B: $(87-100)^2/100 = 169/100 = 1.690$
- Route C: $(134-100)^2/100 = 1156/100 = 11.560$
- Route D: $(96-100)^2/100 = 16/100 = 0.160$
- Route E: $(68-100)^2/100 = 1024/100 = 10.240$
- $\chi^2 = 2.250 + 1.690 + 11.560 + 0.160 + 10.240 = 25.900$
- $df = 5 - 1 = 4$. From chi-square table: $\chi^2_{0.001,4} = 18.467$. Since $25.900 > 18.467$, $p < 0.001$.
- Since $p < 0.001 < \alpha = 0.05$, reject H_0.
- Conclusion: At $\alpha = 0.05$, there is very strong evidence that bus ridership is NOT equally distributed across the five routes. Routes C and E appear to deviate the most from expected.