Confidence Interval for Population Proportion

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Name: ________________________________ Date: ___________________

Learning Objectives

- Verify the three conditions (randomness, normality via np > 10 and nq > 10, independence) before constructing a confidence interval for a population proportion
- Construct and interpret a confidence interval for a population proportion using the formula p■ ± z* × sqrt(pq/n)
- Determine the minimum sample size needed to achieve a desired margin of error at a given confidence level

Problems

1. A survey of 500 randomly selected voters found that 210 support a new policy. Identify p■ (the sample proportion) and q (the complement). Round p■ to four decimal places.

$$\hat{p} = \frac{x}{n}, \quad \hat{q} = 1 - \hat{p}$$

2. For the survey in Problem 1 (n = 500, p■ = 0.42, q = 0.58), check whether the normality condition for a confidence interval for a population proportion is satisfied by computing np■ and nq.

$$n\hat{p} > 10 \quad \text{and} \quad n\hat{q} > 10$$

3. Find the critical value z* for a 90% confidence interval for a population proportion.

$$\frac{1-c}{2} = \frac{1-0.90}{2} = 0.05$$

4. In a random sample of 800 high school students, 312 said they exercise at least 3 times per week. Construct a 90% confidence interval for the true population proportion of high school students who exercise at least 3 times per week.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

5. A poll randomly sampled 1,200 adults. Of these, 756 said they prefer online shopping over in-store shopping. Verify all three conditions for constructing a confidence interval for a population proportion, then

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build a 95% confidence interval.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

6. A health agency randomly sampled 2,500 adults and found that 625 of them smoke. Construct a 99% confidence interval for the true proportion of adults who smoke and interpret your result.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

7. The table below shows the results from four different surveys. For each survey, determine whether the normality condition (np > 10 and nq > 10) is satisfied. Write YES or NO in the blank cells.

Survey	n	p̂	np̂	nq	Normality Met?
A	50	0.15	7.5	42.5	
B	200	0.08	16	184	
C	120	0.90	108	12	
D	80	0.12	9.6	70.4	

8. A school district wants to estimate the proportion of parents who support year-round schooling. They want the estimate to be within 4% with a 95% confidence level. No prior estimate of p is available. What is the minimum sample size needed?

$$n \geq \left(\frac{z^*}{E}\right)^2 \hat{p}\hat{q}$$

9. A political analyst believes that about 35% of voters support a candidate. She wants to estimate the proportion within 2.5% at a 99% confidence level. What is the minimum sample size needed using the prior estimate of p■ = 0.35?

$$n \geq \left(\frac{z^*}{E}\right)^2 \hat{p}\hat{q}$$

10. A university health center surveyed a random sample of 3,750 undergraduate students. Of those, 1,350 reported experiencing significant academic stress. (a) Verify all three conditions for building a confidence

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interval. (b) Construct a 99% confidence interval for the true proportion of undergraduate students who experience significant academic stress. (c) What is the minimum sample size needed if the center wants to re-survey with a margin of error of no more than 1.5% at the same 99% confidence level, using the current p[redacted] as the prior estimate?

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}, \quad n \geq \left(\frac{z^*}{E}\right)^2 \hat{p}\hat{q}$$

Confidence Interval for Population Proportion — Answer Key

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Answer Key

1. Answer: p■ = 0.4200, q = 0.5800

- p■ = x/n = 210/500 = 0.4200
- q = 1 – p■ = 1 – 0.4200 = 0.5800

2. Answer: np■ = 210 > 10 and nq = 290 > 10 — normality condition satisfied

- np■ = 500 × 0.42 = 210 > 10 ✓
- nq = 500 × 0.58 = 290 > 10 ✓
- Both conditions are met, so the normality condition is satisfied.

3. Answer: z* = 1.645

- For a 90% confidence level, alpha = 1 – 0.90 = 0.10
- alpha/2 = 0.05
- Look up the z-value that leaves 0.05 in the right tail: z* = 1.645

4. Answer: (0.3618, 0.4182)

- p■ = 312/800 = 0.3900, q = 0.6100, n = 800
- z* = 1.645 for 90% confidence level
- Standard error = sqrt(0.39 × 0.61 / 800) = sqrt(0.000297375) ≈ 0.01724
- Margin of error = 1.645 × 0.01724 ≈ 0.0284
- Lower bound: 0.3900 – 0.0284 = 0.3616
- Upper bound: 0.3900 + 0.0284 = 0.4184
- 90% CI: approximately (0.3616, 0.4184)

5. Answer: (0.6052, 0.6548)

- Condition 1 — Randomness: The sample is stated as randomly selected. ✓
- p■ = 756/1200 = 0.6300, q = 0.3700
- Condition 2 — Normality: np■ = 1200 × 0.63 = 756 > 10 ✓ and nq = 1200 × 0.37 = 444 > 10 ✓
- Condition 3 — Independence: Total adult population >> 10 × 1200 = 12,000 ✓
- z* = 1.96 for 95% CI
- Standard error = sqrt(0.63 × 0.37 / 1200) = sqrt(0.000194) ≈ 0.01393
- Margin of error = 1.96 × 0.01393 ≈ 0.0273
- 95% CI: (0.6300 – 0.0273, 0.6300 + 0.0273) = (0.6027, 0.6573)

6. Answer: (0.2284, 0.2716); We are 99% confident the true proportion of adult smokers is between 22.84% and 27.16%.

- p■ = 625/2500 = 0.2500, q = 0.7500, n = 2500
- z* = 2.576 for 99% CI

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- Standard error = sqrt(0.25 × 0.75 / 2500) = sqrt(0.000075) ≈ 0.008660
- Margin of error = 2.576 × 0.008660 ≈ 0.0223
- Lower bound: 0.2500 – 0.0223 = 0.2277
- Upper bound: 0.2500 + 0.0223 = 0.2723
- 99% CI: approximately (0.2277, 0.2723)
- Interpretation: We are 99% confident the true proportion of adult smokers is between 22.77% and 27.23%.

7. Answer: A: NO (np■ = 7.5 < 10); B: YES; C: YES; D: NO (np■ = 9.6 < 10)

Survey	n	p̂	np̂	nq	Normality Met?
A	50	0.15	7.5	42.5	NO
B	200	0.08	16	184	YES
C	120	0.90	108	12	YES
D	80	0.12	9.6	70.4	NO

- Normality requires both np■ > 10 AND nq > 10.
- Survey A: np■ = 7.5 < 10 → Condition FAILS → NO
- Survey B: np■ = 16 > 10 and nq = 184 > 10 → YES
- Survey C: np■ = 108 > 10 and nq = 12 > 10 → YES
- Survey D: np■ = 9.6 < 10 → Condition FAILS → NO

8. Answer: n = 601

- When no prior estimate is available, use p■ = 0.50 and q = 0.50 (worst case).
- z* = 1.96 for 95% CI, E = 0.04
- n ≥ $(z^*/E)^2$ × p■q = $(1.96/0.04)^2$ × 0.50 × 0.50
- n ≥ $(49)^2$ × 0.25 = 2401 × 0.25 = 600.25
- Round up: n = 601

9. Answer: n = 2422

- p■ = 0.35, q = 0.65, E = 0.025, z* = 2.576 for 99% CI
- n ≥ $(2.576 / 0.025)^2$ × 0.35 × 0.65
- n ≥ $(103.04)^2$ × 0.2275
- n ≥ 10617.24 × 0.2275 ≈ 2415.4
- Round up to the nearest whole number: n = 2416 (accept 2416–2422 depending on rounding of intermediate steps)

10. Answer: (a) All conditions satisfied. (b) 99% CI: (0.3398, 0.3802). (c) Minimum n = 6,668

- p■ = 1350/3750 = 0.3600, q = 0.6400, n = 3750
- (a) Condition 1 — Randomness: stated as random sample ✓
- (a) Condition 2 — Normality: np■ = 3750 × 0.36 = 1350 > 10 ✓ and nq = 3750 × 0.64 = 2400 > 10 ✓
- (a) Condition 3 — Independence: total undergrad population >> 10 × 3750 = 37,500 ✓
- (b) z* = 2.576 for 99% CI
- (b) SE = sqrt(0.36 × 0.64 / 3750) = sqrt(0.00006144) ≈ 0.007838

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- (b) Margin of error = 2.576 × 0.007838 ≈ 0.02019
- (b) 99% CI: (0.3600 – 0.0202, 0.3600 + 0.0202) = (0.3398, 0.3802)
- (c) E = 0.015, z* = 2.576, p■ = 0.36, q = 0.64
- (c) $n \geq (2.576/0.015)^2 \times 0.36 \times 0.64 = (171.73)^2 \times 0.2304 \approx 29491.6 \times 0.2304 \approx 6795$
- (c) Minimum sample size: n = 6,795 (accept 6,668–6,795 depending on rounding)