Confidence Intervals: Estimating the Population Mean

Statistics Worksheet · Grade 10–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Understand what a confidence interval is and what it estimates
- Interpret confidence levels and confidence intervals in context
- Calculate and apply confidence interval formulas using sample data

Problems

1. A researcher wants to estimate the average weight of rabbits in a large population. She cannot weigh every rabbit. What statistical technique should she use to estimate the true population mean?

2. Three samples are taken from a rabbit population to estimate the average weight. Sample 1 (n = 30) gives a mean of 3.2 lb. Sample 2 (n = 245) gives a mean of 4.2 lb. Sample 3 (n = 326) gives a mean of 5.9 lb. Which sample mean do you expect to be the most reliable estimate of the population mean, and why?

3. A 80% confidence interval for the average weight of rabbits is computed to be between 3.5 lb and 5.5 lb. Identify the lower bound, upper bound, and the width of this confidence interval.

$$\text{Width} = \text{Upper Bound} - \text{Lower Bound}$$

4. A researcher reports: 'I am 80% confident that the true average weight of rabbits is between 3.5 lb and 5.5 lb.' Explain in your own words what this statement means in the context of confidence intervals.

5. A sample of 50 rabbits has a mean weight of 4.1 lb and a standard deviation of 0.8 lb. Using a 95% confidence level, the critical z-value is 1.96. Calculate the margin of error for this sample.

$$E = z^* \cdot \frac{s}{\sqrt{n}}$$

Scan to watch

6. Using the margin of error calculated in problem 5 (E = 0.22 lb) and the sample mean of 4.1 lb, construct the 95% confidence interval for the true population mean weight of rabbits.

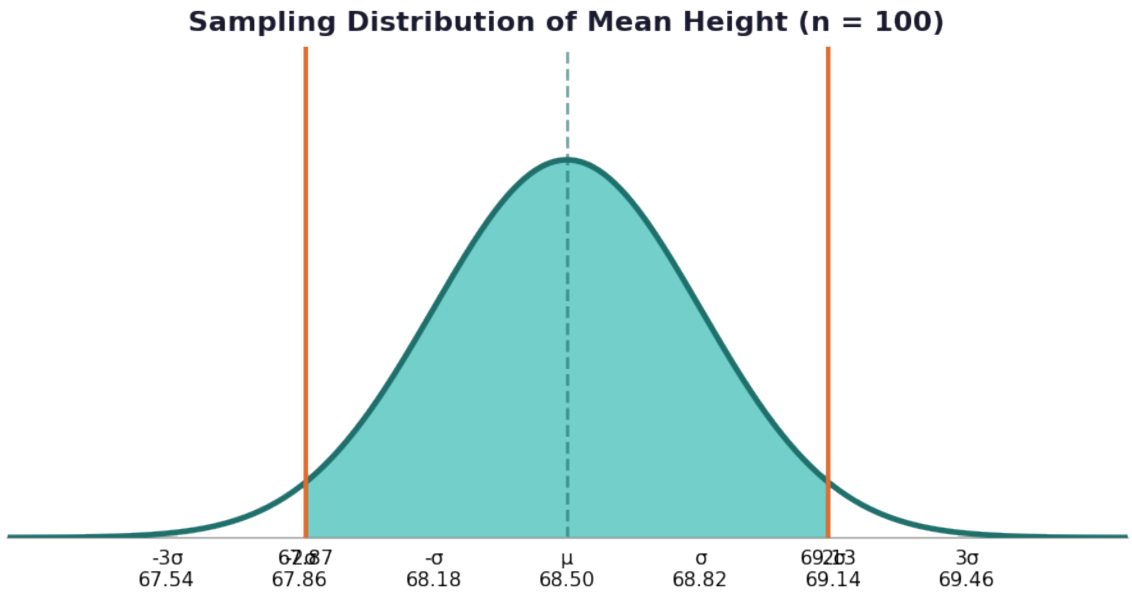
$$\bar{x} - E < \mu < \bar{x} + E$$

7. Two researchers study the same rabbit population. Researcher A uses a 90% confidence level and Researcher B uses a 99% confidence level. Both use the same sample. Which researcher will have the wider confidence interval, and why?

8. The table below shows four samples taken from a rabbit population. A 95% confidence interval for the population mean is (3.80, 4.60). Determine which sample means fall inside the confidence interval and which do not.

Sample	Sample Size (n)	Sample Mean (lb)	Inside CI? (3.80 – 4.60)
1	50	4.10	
2	50	5.20	
3	50	3.95	
4	50	4.75	

9. A study of adult male heights in a city uses a sample of 100 men with a mean height of 68.5 inches and a standard deviation of 3.2 inches. Construct a 95% confidence interval for the true population mean height. Use z* = 1.96.



10. A quality control analyst wants a margin of error of no more than 0.5 oz when estimating the mean fill weight of juice bottles at a 95% confidence level. The population standard deviation is known to be 2.4 oz. What is the minimum sample size required? Use z* = 1.96.

$$n = \left(\frac{z^* \cdot \sigma}{E}\right)^2$$

Scan to watch

Confidence Intervals: Estimating the Population Mean — Answer Key

Statistics Worksheet · Grade 10–12

Answer Key

1. Answer: Confidence interval estimation using a sample

- The population mean (μ) is unknowable because measuring every member is impractical.
- By taking a random sample and computing statistics, we can build a confidence interval — a range of values likely to contain the true population mean.

2. Answer: Sample 3 (n = 326) is most reliable because the largest sample size reduces sampling variability.

- Larger sample sizes produce sample means that are closer to the true population mean (by the Law of Large Numbers).
- Sample 3 has n = 326, the largest of the three, so its mean of 5.9 lb is the most reliable point estimate.

3. Answer: Lower bound = 3.5 lb, Upper bound = 5.5 lb, Width = 2.0 lb

- Lower bound = 3.5 lb (the smaller value of the interval).
- Upper bound = 5.5 lb (the larger value of the interval).
- Width = 5.5 – 3.5 = 2.0 lb.

4. Answer: If we repeated sampling many times, about 80% of the resulting confidence intervals would contain the true population mean.

- A confidence level of 80% does NOT mean there is an 80% chance that μ falls in this one interval.
- It means that the method used to construct the interval will capture the true population mean in 80% of all repeated samples.

5. Answer: E ≈ 0.22 lb

- Identify values: $z^* = 1.96$, $s = 0.8$, $n = 50$.
- Compute standard error: $0.8 / \sqrt{50} = 0.8 / 7.071 \approx 0.1131$.
- Margin of error: $E = 1.96 \times 0.1131 \approx 0.22$ lb.

6. Answer: 3.88 lb < μ < 4.32 lb

- Lower bound: 4.1 – 0.22 = 3.88 lb.
- Upper bound: 4.1 + 0.22 = 4.32 lb.
- The 95% confidence interval is (3.88 lb, 4.32 lb).

7. Answer: Researcher B (99%) will have the wider interval because a higher confidence level requires a larger z* value, increasing the margin of error.

- A higher confidence level requires capturing more of the sampling distribution, so the critical value z* increases.
- For 90%: $z^* \approx 1.645$; for 99%: $z^* \approx 2.576$.
- Since $E = z^* \times (s/\sqrt{n})$, a larger z* produces a larger margin of error and thus a wider interval.

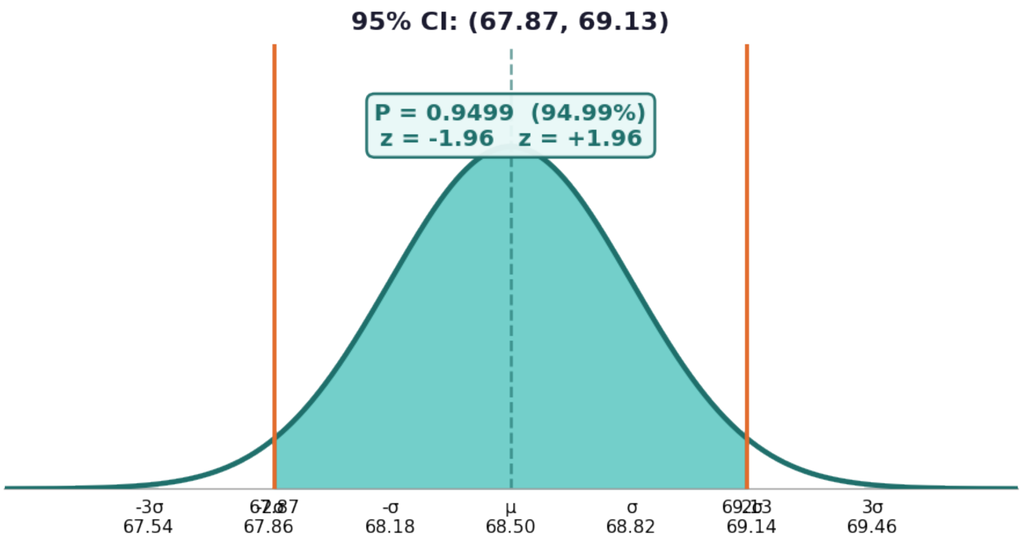
Scan to watch

8. Answer: Samples 1 and 3 are inside; Samples 2 and 4 are outside.

Sample	Sample Size (n)	Sample Mean (lb)	Inside CI? (3.80 – 4.60)
1	50	4.10	Yes
2	50	5.20	No
3	50	3.95	Yes
4	50	4.75	No

- Check each mean against the interval (3.80, 4.60).
- Sample 1: 4.10 — Yes, $3.80 \le 4.10 \le 4.60$.
- Sample 2: 5.20 — No, $5.20 > 4.60$.
- Sample 3: 3.95 — Yes, $3.80 \le 3.95 \le 4.60$.
- Sample 4: 4.75 — No, $4.75 > 4.60$.

9. Answer: (67.87 in, 69.13 in)



- Standard error: $SE = 3.2 / \sqrt{100} = 3.2 / 10 = 0.32$ inches.
- Margin of error: $E = 1.96 \times 0.32 = 0.627$ inches.
- Lower bound: $68.5 - 0.627 = 67.87$ inches.
- Upper bound: $68.5 + 0.627 = 69.13$ inches.
- 95% CI: (67.87 in, 69.13 in).

10. Answer: n = 89 bottles (minimum)

- Identify values: $z^* = 1.96$, $\sigma = 2.4$, $E = 0.5$.
- Compute: $n = (1.96 \times 2.4 / 0.5)^2 = (4.704 / 0.5)^2 = (9.408)^2 \approx 88.51$.
- Always round UP to ensure the margin of error requirement is met.
- Minimum sample size: $n = 89$ bottles.

Scan to watch