Confidence Intervals for Population Mean

Statistics Worksheet · Grade 11–12 / AP Statistics

Name: ________________________________ Date: ___________________

Learning Objectives

- Identify and organize given values (n, x■, σ, confidence level) from a word problem
- Verify the two required conditions for constructing a confidence interval for a population mean
- Compute the critical z-value, margin of error, and confidence interval using the formula x■ ± z* · (σ / √n)

Problems

1. A simple random sample of 100 students from a normally distributed population has a sample mean of 520. The population standard deviation is 80. Identify the values of n, x-bar, and sigma from this problem.

$$n = ?, \quad \bar{x} = ?, \quad \sigma = ?$$

2. A researcher collects a simple random sample of 400 adults from a normally distributed population. State whether the two conditions required for a valid confidence interval for the population mean are satisfied, and explain why.

3. Find the critical z-value (z*) for a 90% confidence interval.

$$z^* = \text{InvNorm}\left(\frac{1 - 0.90}{2}\right)$$

4. Find the critical z-value (z*) for a 99% confidence interval.

$$z^* = \text{InvNorm}\left(\frac{1 - 0.99}{2}\right)$$

5. A random sample of 64 light bulbs is taken from a normally distributed population. The sample mean life is 1200 hours and the population standard deviation is 96 hours. Calculate the margin of error for a 95% confidence interval.

$$E = z^* \cdot \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{96}{\sqrt{64}}$$

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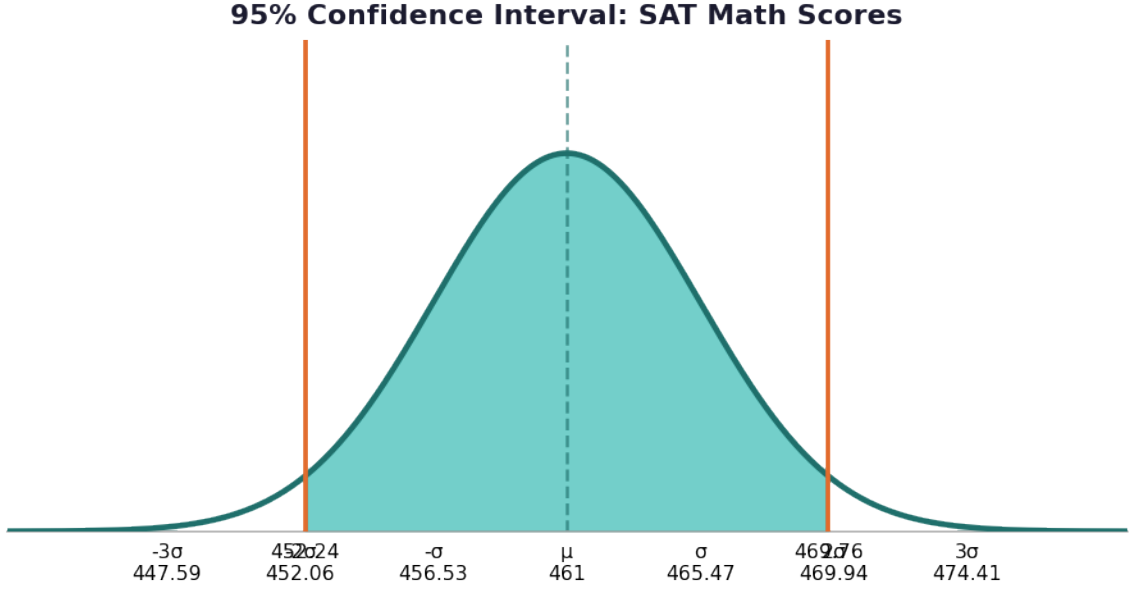
6. Using the information from Problem 5 (sample mean = 1200, margin of error = 23.52), construct the 95% confidence interval for the population mean life of the light bulbs.

$$\bar{x} \pm E = 1200 \pm 23.52$$

7. From a normally distributed population, an SRS of 500 students had a mean SAT math score of 461 with a population standard deviation of 100. Construct the full 95% confidence interval and shade the corresponding region on the normal curve shown below.



8. A study samples 225 college athletes from a normally distributed population. The sample mean sprint time is 11.4 seconds and the population standard deviation is 1.5 seconds. Construct a 99% confidence interval for the true population mean sprint time.

$$\bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}} = 11.4 \pm 2.576 \cdot \frac{1.5}{\sqrt{225}}$$

9. A researcher wants to estimate the population mean with a margin of error of no more than 5 points at the 95% confidence level. The population standard deviation is known to be 50. What is the minimum sample size needed?

$$n \geq \left(\frac{z^* \cdot \sigma}{E}\right)^2 = \left(\frac{1.96 \times 50}{5}\right)^2$$

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10. Two researchers each draw a separate SRS from the same normally distributed population with sigma = 120. Researcher A uses n = 100 and gets a sample mean of 540. Researcher B uses n = 900 and gets a sample mean of 534. Both construct 95% confidence intervals. Which researcher has the narrower interval, and by how much is the margin of error reduced? Write both intervals and compare.

$$E_A = 1.96 \cdot \frac{120}{\sqrt{100}}, \quad E_B = 1.96 \cdot \frac{120}{\sqrt{900}}$$

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Confidence Intervals for Population Mean — Answer Key

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Answer Key

1. Answer: $n = 100$, x■ = 520, $\sigma = 80$

- n is the sample size: $n = 100$
- x■ is the sample mean: x■ = 520
- σ is the population standard deviation: $\sigma = 80$

2. Answer: Yes — (1) SRS satisfies random selection; (2) normally distributed population satisfies the normality condition.

- Condition 1: The sample must be randomly selected. It is an SRS, so this condition is met.
- Condition 2: The population must be normally distributed (or $n \geq 30$ by CLT). The problem states it is normally distributed, so this condition is met.
- Both conditions are satisfied, so inference is valid.

3. Answer: $z^* = 1.645$

- Use the formula: find the tail area = $(1 – 0.90) / 2 = 0.05$
- Look up the z-value with a right-tail area of 0.05 (or left-tail area of 0.95) in the z-table.
- $z^* = 1.645$

4. Answer: $z^* = 2.576$

- Tail area = $(1 – 0.99) / 2 = 0.005$
- Look up z corresponding to a cumulative left-tail probability of 0.995.
- $z^* = 2.576$

5. Answer: $E = 23.52$ hours

- z^* for 95% confidence = 1.96
- Standard error = $\sigma / \sqrt{n} = 96 / \sqrt{64} = 96 / 8 = 12$
- Margin of error $E = 1.96 \times 12 = 23.52$ hours

6. Answer: (1176.48, 1223.52) hours

- Lower bound = $1200 – 23.52 = 1176.48$
- Upper bound = $1200 + 23.52 = 1223.52$
- 95% CI: (1176.48, 1223.52) hours

7. Answer: (452.24, 469.76)

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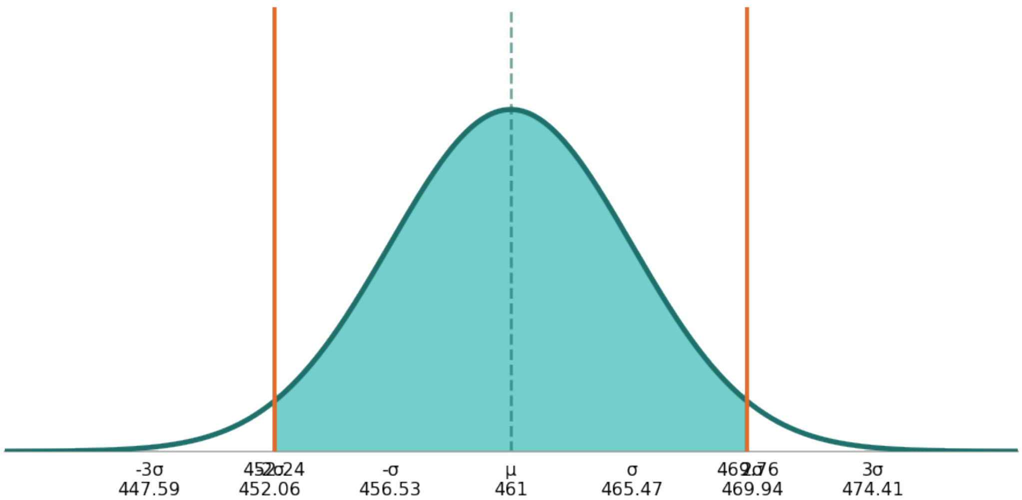

- $z^* = 1.96$ for 95% confidence
- Standard error = 100 / √500 = 100 / 22.36 ≈ 4.47
- Margin of error = 1.96 × 4.47 ≈ 8.76
- Lower bound = 461 – 8.76 = 452.24
- Upper bound = 461 + 8.76 = 469.76
- 95% CI: (452.24, 469.76)

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8. Answer: (11.143, 11.657) seconds

- $z^* = 2.576$ for 99% confidence
- Standard error = 1.5 / √225 = 1.5 / 15 = 0.10
- Margin of error = 2.576 × 0.10 = 0.2576 ≈ 0.257
- Lower bound = 11.4 – 0.257 = 11.143
- Upper bound = 11.4 + 0.257 = 11.657
- 99% CI: (11.143, 11.657) seconds

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9. Answer: n ≥ 385

- Use the minimum sample size formula: $n \geq (z^* \cdot \sigma / E)^2$
- $z^* = 1.96$, $\sigma = 50$, $E = 5$
- $n \geq (1.96 \times 50 / 5)^2 = (19.6)^2 = 384.16$
- Round up to the nearest whole number: $n \geq 385$

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10. Answer: Researcher B has the narrower interval. E_A = 23.52, E_B = 7.84. CI_A: (516.48, 563.52); CI_B: (526.16, 541.84). The margin of error is reduced by 15.68 points.

- Researcher A: SE = 120 / √100 = 12; E_A = 1.96 × 12 = 23.52; CI: (540 – 23.52, 540 + 23.52) = (516.48, 563.52)
- Researcher B: SE = 120 / √900 = 4; E_B = 1.96 × 4 = 7.84; CI: (534 – 7.84, 534 + 7.84) = (526.16, 541.84)
- Researcher B's interval is narrower because the larger sample size reduces the standard error.
- Reduction in margin of error = 23.52 – 7.84 = 15.68 points
- Conclusion: Increasing sample size from 100 to 900 (a factor of 9) reduces the margin of error by a factor of 3 (= √9).

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