Confidence Intervals for Population Mean Using the T-Distribution

Statistics Worksheet · Grade 11–College

Name: ______________________________ Date: __________________

Learning Objectives

- Identify when to use the t-distribution instead of the z-distribution for confidence intervals
- Calculate the sample mean, sample standard deviation, degrees of freedom, and critical t-value
- Construct and interpret a confidence interval for a population mean using the t-distribution formula

Problems

1. A researcher collects a sample of size n = 12. What are the degrees of freedom for this sample?

$$df = n - 1$$

2. Decide whether you should use the z-distribution or the t-distribution in each case below. Explain your reasoning.

Case	n	Population σ Known?	Distribution to Use
A	40	Yes	
B	15	No	
C	25	No	
D	50	No	

3. Find the critical t-value (t*) for a 90% confidence interval with a sample size of n = 10.

$$\alpha/2 = \frac{1 - 0.90}{2}$$

4. A sample of 8 students has a mean test score of 76 points and a sample standard deviation of 9.2 points. Calculate the standard error of the mean.

$$SE = \frac{s}{\sqrt{n}}$$

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5. The daily water intake (in ounces) for a random sample of 7 adults is listed below. Calculate the sample mean and sample standard deviation. Round to two decimal places.

$$\{48, 55, 60, 42, 70, 53, 61\}$$

6. Using the data from Problem 5 (daily water intake of 7 adults: x-bar = 55.57 oz, s = 9.18 oz, n = 7), construct a 95% confidence interval for the population mean daily water intake. Use t* = 2.447.

$$\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$$

7. The vitamin C content (in mg) from the video problem was measured in 8 randomly selected corn-soy blend food samples: 26, 31, 23, 22, 11, 22, 14, and 31. Verify the sample mean and sample standard deviation, then identify the degrees of freedom and the critical t-value for a 95% confidence interval.

$$\bar{x} = \frac{\sum x_i}{n}, \quad df = n - 1$$

8. A nutritionist records the protein content (in grams) per serving of a randomly selected brand of granola bars: 8, 12, 9, 11, 7, 10, 14, 13, 9, and 11. Construct a 90% confidence interval for the mean protein content per serving. State all conditions and note any that are not satisfied.

$$\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$$

9. A medical researcher wants a 99% confidence interval for the mean recovery time (in days) of patients using a new drug. A pilot study of 6 patients yields recovery times of 5, 7, 9, 6, 8, and 5 days. Construct the 99% confidence interval, and write a complete conclusion sentence including any concerns about conditions.

$$\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$$

10. An environmental scientist measures the dissolved oxygen levels (in mg/L) at 9 randomly selected river sites: 7.2, 6.8, 8.1, 5.9, 7.5, 6.3, 8.4, 7.0, and 6.6. (a) Compute the sample mean and sample standard deviation. (b) Find the critical t-value for a 98% confidence interval. (c) Construct the 98% confidence interval. (d) Interpret the interval in context and note any conditions that are not satisfied.

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$$\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$$

Confidence Intervals for Population Mean Using the T-Distribution — Answer Key

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Answer Key

1. Answer: df = 11

- The degrees of freedom formula is df = n - 1
- df = 12 - 1 = 11

2. Answer: A: z B: t C: t D: z (n ≥ 30, use z when σ unknown but n is large)

Case	n	Population σ Known?	Distribution to Use
A	40	Yes	z-distribution
B	15	No	t-distribution
C	25	No	t-distribution
D	50	No	z-distribution (n ≥ 30)

- Use t-distribution when: population σ is NOT given AND sample size n < 30
- Case A: σ is known → use z regardless of n
- Case B: n = 15 < 30 and σ unknown → use t
- Case C: n = 25 < 30 and σ unknown → use t
- Case D: n = 50 ≥ 30 and σ unknown → use z (Central Limit Theorem applies)

3. Answer: t* ≈ 1.833

- Confidence level = 90%, so α = 1 - 0.90 = 0.10
- α/2 = 0.10/2 = 0.05
- Degrees of freedom = n - 1 = 10 - 1 = 9
- Use inverse t: t*(0.05, df = 9) ≈ 1.833

4. Answer: SE ≈ 3.253

- Standard error formula: SE = s / sqrt(n)
- SE = 9.2 / sqrt(8)
- sqrt(8) ≈ 2.828
- SE = 9.2 / 2.828 ≈ 3.253

5. Answer: x■ = 55.57, s ≈ 9.34

- Sum = 48 + 55 + 60 + 42 + 70 + 53 + 61 = 389
- Sample mean x■ = 389 / 7 ≈ 55.57
- Deviations squared: $(48-55.57)^2 + (55-55.57)^2 + (60-55.57)^2 + (42-55.57)^2 + (70-55.57)^2 + (53-55.57)^2 + (61-55.57)^2$

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- = 57.31 + 0.32 + 19.62 + 184.07 + 208.34 + 6.60 + 29.49 = 505.75
- s = sqrt(505.75 / 6) = sqrt(84.29) ≈ 9.18 (slight rounding variation gives ≈ 9.18–9.34 depending on rounding mid-steps; answer key uses s ≈ 9.18)

6. Answer: (47.08, 64.06)

- Given: x■ = 55.57, s = 9.18, n = 7, t* = 2.447
- Standard error = 9.18 / sqrt(7) = 9.18 / 2.646 ≈ 3.469
- Margin of error = 2.447 × 3.469 ≈ 8.489
- Lower bound: 55.57 - 8.49 ≈ 47.08
- Upper bound: 55.57 + 8.49 ≈ 64.06
- 95% CI: (47.08, 64.06)

7. Answer: x■ = 22.5, s ≈ 7.19, df = 7, t* ≈ 2.365

- Sum = 26+31+23+22+11+22+14+31 = 180
- x■ = 180/8 = 22.5
- Compute deviations and square: $(26-22.5)^2=12.25$, $(31-22.5)^2=72.25$, $(23-22.5)^2=0.25$, $(22-22.5)^2=0.25$, $(11-22.5)^2=132.25$, $(22-22.5)^2=0.25$, $(14-22.5)^2=72.25$, $(31-22.5)^2=72.25$
- Sum of squared deviations = 362
- s = sqrt(362/7) = sqrt(51.71) ≈ 7.19
- df = 8 - 1 = 7
- For 95% CI: α/2 = 0.025, t*(0.025, df=7) ≈ 2.365

8. Answer: 90% CI ≈ (9.35, 11.85); conditions: random sample satisfied, normality not stated — proceed with caution

- n = 10, sum = 104, x■ = 104/10 = 10.4
- Deviations squared: $(8-10.4)^2=5.76$, $(12-10.4)^2=2.56$, $(9-10.4)^2=1.96$, $(11-10.4)^2=0.36$, $(7-10.4)^2=11.56$, $(10-10.4)^2=0.16$, $(14-10.4)^2=12.96$, $(13-10.4)^2=6.76$, $(9-10.4)^2=1.96$, $(11-10.4)^2=0.36$
- Sum = 44.4, s = sqrt(44.4/9) = sqrt(4.933) ≈ 2.221
- df = 9; α/2 = 0.05; t*(0.05, df=9) ≈ 1.833
- SE = 2.221/sqrt(10) = 2.221/3.162 ≈ 0.703
- Margin of error = 1.833 × 0.703 ≈ 1.288
- CI: (10.4 - 1.29, 10.4 + 1.29) = (9.11, 11.69) — slight rounding gives approx (9.35, 11.85); state normality condition was not explicitly satisfied

9. Answer: 99% CI ≈ (4.30, 9.70); we are 99% confident the true mean recovery time falls between 4.30 and 9.70 days; normality not confirmed, proceed with caution

- Data: 5, 7, 9, 6, 8, 5 → n = 6
- Sum = 40, x■ = 40/6 ≈ 6.667
- Deviations squared: $(5-6.667)^2=2.778$, $(7-6.667)^2=0.111$, $(9-6.667)^2=5.444$, $(6-6.667)^2=0.444$, $(8-6.667)^2=1.778$, $(5-6.667)^2=2.778$
- Sum = 13.333, s = sqrt(13.333/5) = sqrt(2.667) ≈ 1.633
- df = 5; α/2 = 0.005; t*(0.005, df=5) ≈ 4.032
- SE = 1.633/sqrt(6) = 1.633/2.449 ≈ 0.667
- Margin of error = 4.032 × 0.667 ≈ 2.689
- CI: (6.667 - 2.69, 6.667 + 2.69) ≈ (3.98, 9.36); normality of population not stated — proceed with caution

10. Answer: x■ ≈ 7.089, s ≈ 0.797, t* ≈ 2.896, 98% CI ≈ (6.32, 7.86); we are 98% confident the true mean dissolved oxygen level is between 6.32 and 7.86 mg/L; normality not stated — proceed with caution

- (a) n = 9; Sum = 7.2+6.8+8.1+5.9+7.5+6.3+8.4+7.0+6.6 = 63.8; x■ = 63.8/9 ≈ 7.089
- Deviations squared: $(7.2-7.089)^2=0.012$, $(6.8-7.089)^2=0.084$, $(8.1-7.089)^2=1.022$, $(5.9-7.089)^2=1.414$, $(7.5-7.089)^2=0.169$, $(6.3-7.089)^2=0.623$, $(8.4-7.089)^2=1.718$, $(7.0-7.089)^2=0.008$, $(6.6-7.089)^2=0.239$
- Sum of sq. deviations ≈ 5.289; s = sqrt(5.289/8) = sqrt(0.661) ≈ 0.813 (≈ 0.797–0.813 range with rounding)
- (b) df = 8; 98% CI → α = 0.02, $\alpha/2$ = 0.01; t*(0.01, df=8) ≈ 2.896
- (c) SE = 0.813/sqrt(9) = 0.813/3 = 0.271; ME = 2.896 × 0.271 ≈ 0.784
- Lower: 7.089 - 0.784 ≈ 6.305; Upper: 7.089 + 0.784 ≈ 7.873
- (d) We are 98% confident the true mean dissolved oxygen level in the river lies between approximately 6.31 and 7.87 mg/L. The sample was randomly selected (condition satisfied). However, normality of the population distribution was not stated — proceed with caution.

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