Confidence Intervals for Population Proportion

Statistics Worksheet · Grade 11–12 / AP Statistics

Name: ________________________________ Date: __________________

Learning Objectives

- Identify the population and parameter of interest in a real-world context involving proportions
- Verify the three conditions (randomness, independence, normality) required to construct a confidence interval for a population proportion
- Construct and interpret confidence intervals for a population proportion, and determine the required sample size given a margin of error and confidence level

Problems

1. A survey asked 200 randomly selected high school students whether they own a smartphone. Identify the population and the parameter of interest.

2. In a random sample of 150 registered voters, 93 said they plan to vote in the next election. Calculate the sample proportion p-hat and its complement q-hat.

$$\hat{p} = \frac{x}{n}, \quad \hat{q} = 1 - \hat{p}$$

3. A researcher takes a simple random sample of 180 college students from a large university with over 20,000 students enrolled. She finds that 45 students work more than 20 hours per week. Check whether all three conditions for constructing a confidence interval for a proportion are satisfied.

$$\hat{p} = \frac{45}{180} = 0.25$$

4. Find the critical value z* for each confidence level: (a) 90%, (b) 95%, (c) 99%.

$$\alpha = 1 - C, \quad z^* = z_{\alpha/2}$$

5. A poll of 250 randomly selected adults from a city of 500,000 found that 110 support a new public transit plan. Construct a 95% confidence interval for the true proportion of adults who support the plan.

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$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

6. A 95% confidence interval for the proportion of students who prefer online classes was reported as (0.42, 0.58). Identify the sample proportion p-hat, the margin of error, and interpret the interval in context.

$$\hat{p} = \frac{\text{lower} + \text{upper}}{2}, \quad ME = \frac{\text{upper} - \text{lower}}{2}$$

7. A university wants to estimate the proportion of students who have used the campus tutoring center. With no prior estimate available, how large a sample is needed to be within 0.05 of the true proportion at a 99% confidence level? Use the conservative estimate for p-hat.

$$n = \left(\frac{z^*}{ME}\right)^2 \hat{p}\hat{q}$$

8. A previous study found that 68% of teenagers use social media daily. A researcher wants to estimate the current proportion with a margin of error of 0.03 at the 90% confidence level. What is the minimum sample size needed?

$$n = \left(\frac{z^*}{ME}\right)^2 \hat{p}\hat{q}$$

9. In a survey of 320 randomly selected employees at a large company, 80 said they experienced burnout in the past year. Construct a 99% confidence interval for the true proportion. Then determine whether the data support the company's claim that fewer than 30% of employees experience burnout.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

10. Two independent surveys were conducted to compare the proportions of adults who recycle regularly. Survey A sampled 400 adults and found 52% recycle. Survey B sampled 500 adults and found 45% recycle. Construct separate 95% confidence intervals for each group, compare them, and comment on whether there is convincing evidence of a difference in the true proportions.

$$\hat{p}_A \pm z^* \sqrt{\frac{\hat{p}_A\hat{q}_A}{n_A}}, \quad \hat{p}_B \pm z^* \sqrt{\frac{\hat{p}_B\hat{q}_B}{n_B}}$$

Confidence Intervals for Population Proportion — Answer Key

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Answer Key

1. Answer: Population: all high school students. Parameter of interest: the proportion of high school students who own a smartphone.

- The population is the entire group being studied: all high school students.
- The parameter of interest is the numerical characteristic we want to estimate: the true proportion (p) of high school students who own a smartphone.
- The 200 surveyed students are the sample, not the population.

2. Answer: p-hat = 0.62, q-hat = 0.38

- p-hat = x / n = 93 / 150 = 0.62
- q-hat = 1 – p-hat = 1 – 0.62 = 0.38

3. Answer: All three conditions are satisfied: random (SRS stated), independent (20,000 > 10 × 180 = 1,800), normal (np = 45 ≥ 10 and nq = 135 ≥ 10).

- Condition 1 – Randomness: The sample is described as an SRS, so this condition is met.
- Condition 2 – Independence: The university has over 20,000 students. Check: N > 10n → 20,000 > 10(180) = 1,800. ✓
- Condition 3 – Normality: p-hat = 45/180 = 0.25, q-hat = 0.75. Check np = 180(0.25) = 45 ≥ 10 ✓ and nq = 180(0.75) = 135 ≥ 10 ✓.
- All three conditions are satisfied. It is safe to proceed with a confidence interval.

4. Answer: (a) z* = 1.645, (b) z* = 1.960, (c) z* = 2.576

- (a) 90% CI: $\alpha = 0.10$, $\alpha/2 = 0.05$. Using the standard normal table, $z^* = 1.645$.
- (b) 95% CI: $\alpha = 0.05$, $\alpha/2 = 0.025$. $z^* = 1.960$.
- (c) 99% CI: $\alpha = 0.01$, $\alpha/2 = 0.005$. $z^* = 2.576$.

5. Answer: (0.378, 0.502)

- p-hat = 110/250 = 0.44, q-hat = 0.56, n = 250, z* = 1.960.
- Check conditions: SRS ✓; 500,000 > 10(250) = 2,500 ✓; np = 110 ≥ 10 and nq = 140 ≥ 10 ✓.
- Margin of error = $1.960 \times \sqrt{(0.44 \times 0.56 / 250)} = 1.960 \times \sqrt{(0.0009856)} = 1.960 \times 0.03139 \approx 0.0615$.
- CI: 0.44 ± 0.0615 → (0.378, 0.502).
- We are 95% confident the true proportion of adults who support the plan is between 37.8% and 50.2%.

6. Answer: p-hat = 0.50; ME = 0.08; We are 95% confident the true proportion who prefer online classes is between 42% and 58%.

- p-hat = (0.42 + 0.58) / 2 = 1.00 / 2 = 0.50.
- Margin of error = (0.58 – 0.42) / 2 = 0.16 / 2 = 0.08.

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• Interpretation: We are 95% confident that the true proportion of all students who prefer online classes lies between 0.42 and 0.58.
• Note: '95% confident' means that if we repeated this sampling process many times, about 95% of the resulting intervals would contain the true population proportion.

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7. Answer: n = 664

• When no prior estimate is given, use the conservative estimate: p-hat = 0.50, q-hat = 0.50.
• z^* for 99% CI = 2.576. ME = 0.05.
• $n = (z^*/ME)^2 \times$ p-hat $\times$ q-hat $= (2.576/0.05)^2 \times 0.50 \times 0.50$.
• $n = (51.52)^2 \times 0.25 = 2{,}654.31 \times 0.25 = 663.58$.
• Always round UP: n = 664.

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8. Answer: n = 655

• p-hat = 0.68, q-hat = 0.32 (from prior study).
• z^* for 90% CI = 1.645. ME = 0.03.
• $n = (1.645/0.03)^2 \times 0.68 \times 0.32$.
• $n = (54.833)^2 \times 0.2176 = 3{,}006.67 \times 0.2176 \approx 654.25$.
• Round up: n = 655.

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9. Answer: CI: (0.187, 0.313). The claim is not clearly supported because 0.30 is inside the interval.

• p-hat = 80/320 = 0.25, q-hat = 0.75, n = 320, $z^* = 2.576$.
• Check conditions: SRS assumed ✓; large company N > 10(320) ✓; np = 80 ≥ 10, nq = 240 ≥ 10 ✓.
• $ME = 2.576 \times \sqrt{(0.25 \times 0.75 / 320)} = 2.576 \times \sqrt{(0.000586)} = 2.576 \times 0.02420 \approx 0.0623$.
• CI: 0.25 ± 0.0623 → (0.1877, 0.3123).
• Since 0.30 falls inside the interval (0.188, 0.312), we cannot rule out that the true proportion equals or exceeds 30%. The company's claim of 'fewer than 30%' is not conclusively supported.

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10. Answer: CI_A: (0.471, 0.569); CI_B: (0.406, 0.494). The intervals overlap slightly, so there is not convincing evidence of a difference at the 95% confidence level.

• Survey A: p-hat_A = 0.52, q-hat_A = 0.48, n_A = 400. $ME_A = 1.960 \times \sqrt{(0.52 \times 0.48/400)} = 1.960 \times 0.02499 \approx 0.049$. CI_A: (0.471, 0.569).
• Survey B: p-hat_B = 0.45, q-hat_B = 0.55, n_B = 500. $ME_B = 1.960 \times \sqrt{(0.45 \times 0.55/500)} = 1.960 \times 0.02225 \approx 0.044$. CI_B: (0.406, 0.494).
• CI_A = (0.471, 0.569) and CI_B = (0.406, 0.494) overlap in the range (0.471, 0.494).
• Because the intervals overlap, at the 95% confidence level we do not have convincing evidence that the true proportions differ.
• Note: Overlapping confidence intervals do not definitively prove the proportions are equal — a formal two-proportion z-test would be needed for a definitive conclusion.

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