Confidence Intervals for the Population Mean

Statistics Worksheet · Grade 11–12 / AP Statistics

Name: ______________________________ Date: __________________

Learning Objectives

- Identify and organize given values (n, $\bar{x}$, σ, confidence level) from a word problem
- Verify the two conditions required for a valid confidence interval (random sample, normal population or large n)
- Calculate the margin of error and confidence interval using the formula $\bar{x}$ ± z* · (σ / √n)

Problems

1. A simple random sample of 100 students has a mean SAT math score of 520. The population standard deviation is known to be 80. Identify and list all four given values needed to construct a confidence interval.

$$n,\ \bar{x},\ \sigma,\ \text{confidence level}$$

2. State the two conditions that must be verified before constructing a confidence interval for a population mean, and explain why each matters.

3. Find the critical value z* for a 90% confidence interval using the formula shown. Then find z* for a 99% confidence interval.

$$z^*\ \text{where}\ \alpha/2 = \frac{1-C}{2}$$

4. From a normally distributed population, a simple random sample of 64 observations has a mean of 72. The population standard deviation is 16. Calculate the standard error of the mean.

$$SE = \frac{\sigma}{\sqrt{n}}$$

5. Using the sample in Problem 4 (n = 64, $\bar{x}$ = 72, σ = 16), construct the 95% confidence interval for the population mean. Use z* = 1.96.

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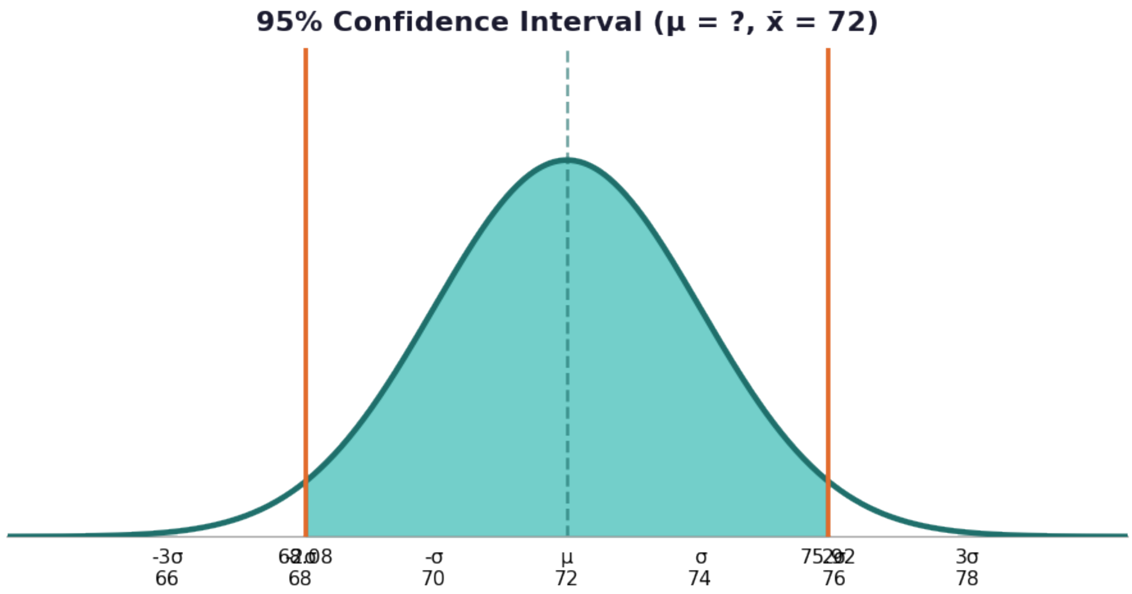
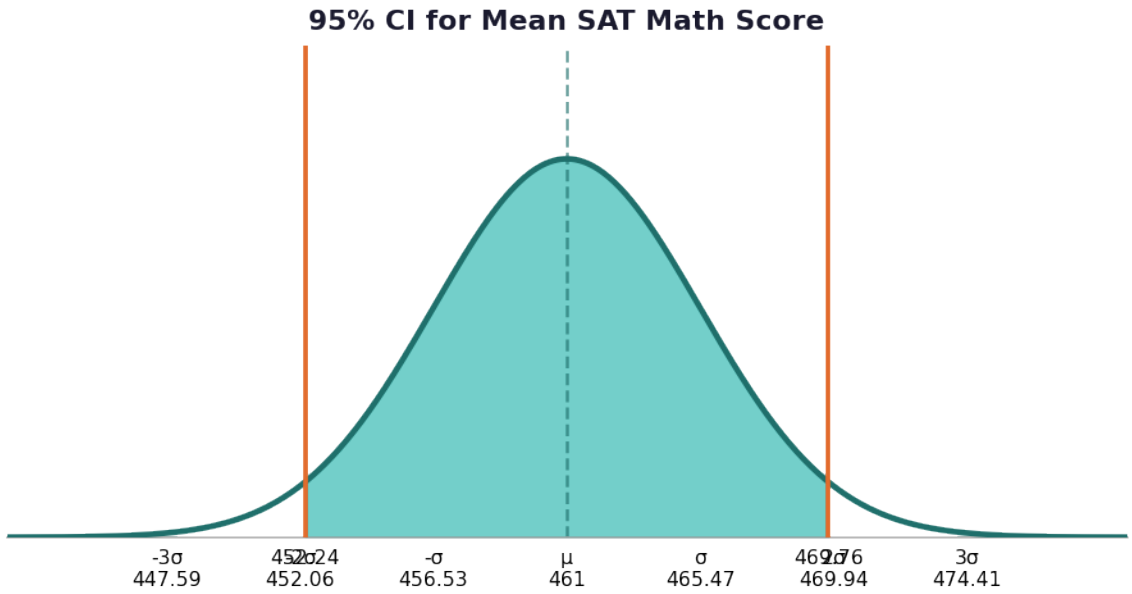


6. A researcher takes an SRS of 500 students and finds a mean SAT math score of 461. The population standard deviation is 100. Calculate the margin of error for a 95% confidence interval. Use z* = 1.96.

$$ME = z^* \cdot \frac{\sigma}{\sqrt{n}}$$

7. Using the data from Problem 6 ($n = 500$, $\bar{x} = 461$, $\sigma = 100$), construct the full 95% confidence interval and write a sentence interpreting the result in context.



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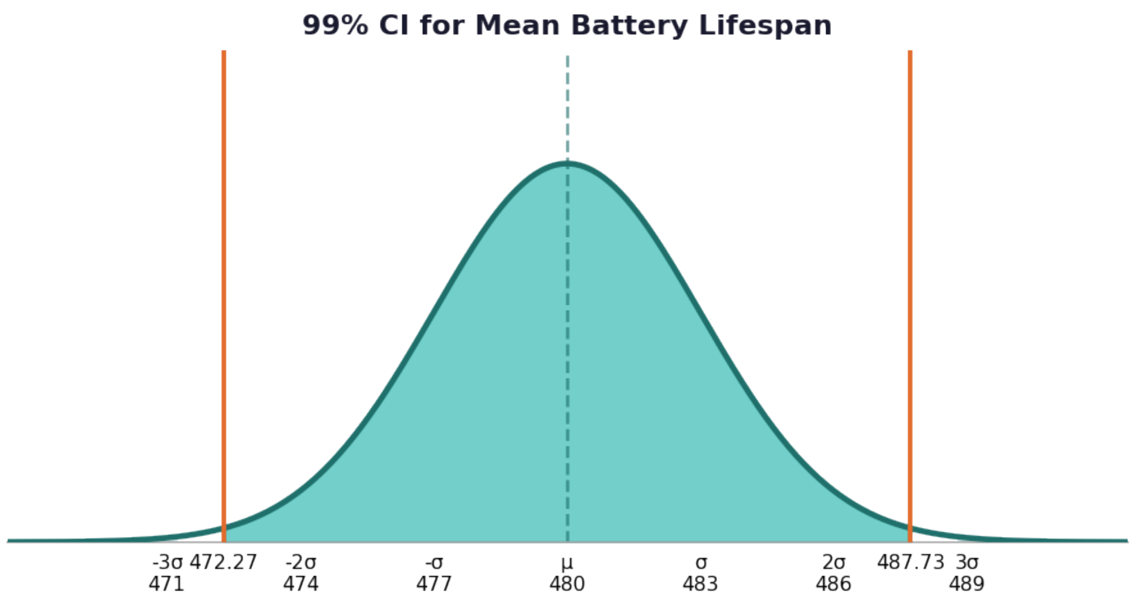
8. A normally distributed population has a standard deviation of 25. How large a simple random sample is needed so that the margin of error for a 95% confidence interval is no more than 5 points? Use z* = 1.96.

$$n \geq \left(\frac{z^* \cdot \sigma}{ME}\right)^2$$

9. Two students each take a simple random sample from the same normally distributed population (σ = 50). Student A uses n = 100 and gets x■ = 310. Student B uses n = 400 and gets x■ = 310. Both construct 95% confidence intervals. Find both intervals and explain which is narrower and why.

$$\bar{x} \pm 1.96 \cdot \frac{50}{\sqrt{n}}$$

10. A quality-control engineer takes an SRS of 225 batteries from a normally distributed production line and records a mean lifespan of 480 hours. The population standard deviation is 45 hours. Construct the 99% confidence interval for the true mean lifespan, verify both conditions, and write a conclusion statement. Use z* = 2.576.



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Confidence Intervals for the Population Mean — Answer Key

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Answer Key

1. Answer: $n = 100$, x■ $= 520$, $\sigma = 80$, $C = 0.95$ (assumed standard)

- Step 1: Sample size $n = 100$ (from 'SRS of 100 students').
- Step 2: Sample mean x■ $= 520$ (from the SRS).
- Step 3: Population standard deviation $\sigma = 80$ (given).
- Step 4: Confidence level $C = 0.95$ (stated or assumed).

2. Answer: 1) Sample is randomly selected. 2) Population is normally distributed (or $n \geq 30$ by CLT).

- Condition 1: The sample must be randomly selected so results are unbiased and representative of the population.
- Condition 2: The population must be normally distributed (or the sample size must be large enough for the Central Limit Theorem to apply) so the sampling distribution of x■ is normal.

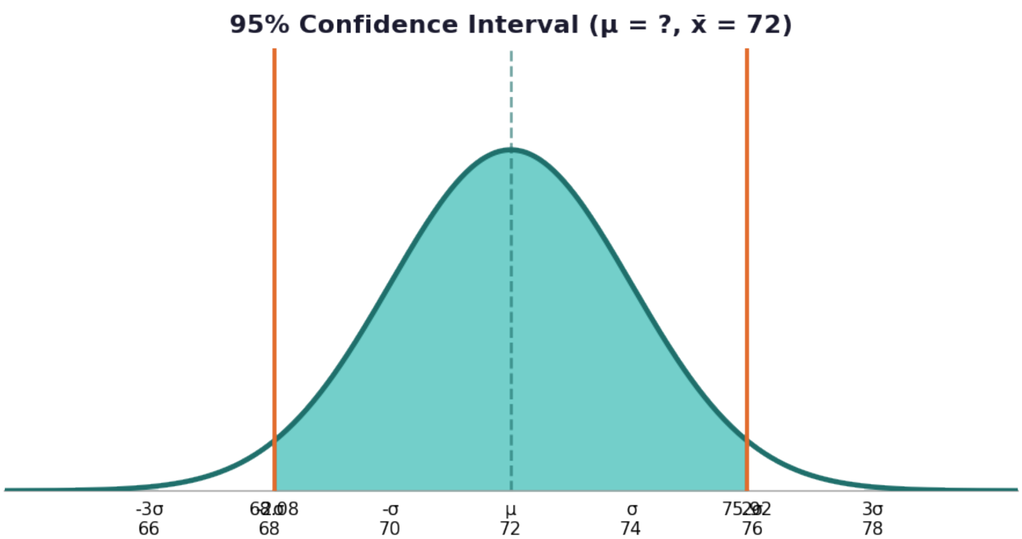
3. Answer: z^* for 90% = 1.645; z^* for 99% = 2.576

- For 90%: $\alpha/2 = (1 - 0.90)/2 = 0.05$. Look up z with area 0.95 in the z-table → $z^* = 1.645$.
- For 99%: $\alpha/2 = (1 - 0.99)/2 = 0.005$. Look up z with area 0.995 in the z-table → $z^* = 2.576$.

4. Answer: SE = 2

- Step 1: Identify $\sigma = 16$ and $n = 64$.
- Step 2: $SE = \sigma / \sqrt{n} = 16 / \sqrt{64} = 16 / 8 = 2$.

5. Answer: 95% CI: (68.08, 75.92)



- Step 1: $SE = 16 / \sqrt{64} = 2$.

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- Step 2: Margin of error = z* × SE = 1.96 × 2 = 3.92.
- Step 3: Lower bound = 72 – 3.92 = 68.08.
- Step 4: Upper bound = 72 + 3.92 = 75.92.
- Step 5: 95% CI: (68.08, 75.92).

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6. Answer: ME ≈ 8.76

- Step 1: SE = σ / √n = 100 / √500 ≈ 100 / 22.36 ≈ 4.47.
- Step 2: ME = z* × SE = 1.96 × 4.47 ≈ 8.76.

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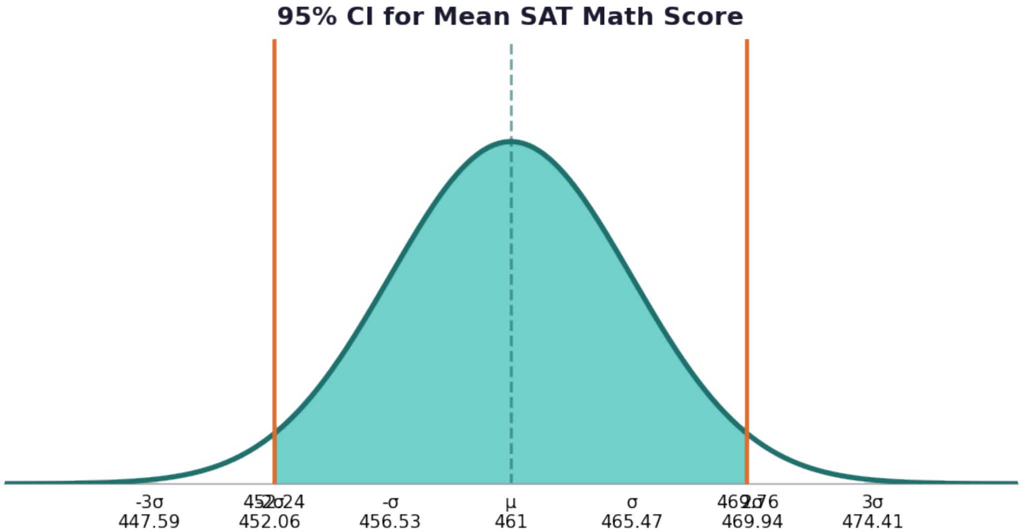
7. Answer: 95% CI: (452.24, 469.76). We are 95% confident the true mean SAT math score lies between 452.24 and 469.76.



- Step 1: ME = 1.96 × (100 / √500) ≈ 1.96 × 4.47 ≈ 8.76.
- Step 2: Lower bound = 461 – 8.76 = 452.24.
- Step 3: Upper bound = 461 + 8.76 = 469.76.
- Step 4: Interpretation: We are 95% confident the true population mean SAT math score lies between 452.24 and 469.76 points.

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8. Answer: n ≥ 97 (round up to 97)

- Step 1: Set up the inequality: ME ≤ 5, so z* · σ / √n ≤ 5.
- Step 2: Solve for n: √n ≥ (1.96 × 25) / 5 = 49 / 5 = 9.8.
- Step 3: $n \geq 9.8^2 = 96.04$.
- Step 4: Round up to the nearest whole number: n ≥ 97.

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9. Answer: Student A: (300.2, 319.8); Student B: (305.1, 314.9). Student B's interval is narrower because larger n reduces SE.

- Student A: SE = 50 / √100 = 5; ME = 1.96 × 5 = 9.8; CI = (310 – 9.8, 310 + 9.8) = (300.2, 319.8).
- Student B: SE = 50 / √400 = 2.5; ME = 1.96 × 2.5 = 4.9; CI = (310 – 4.9, 310 + 4.9) = (305.1, 314.9).
- Student B's interval is narrower because a larger sample size reduces the standard error, which reduces the margin of error.
- Both intervals are centered at x■ = 310, but Student B's width is only 9.8 vs. Student A's 19.6.

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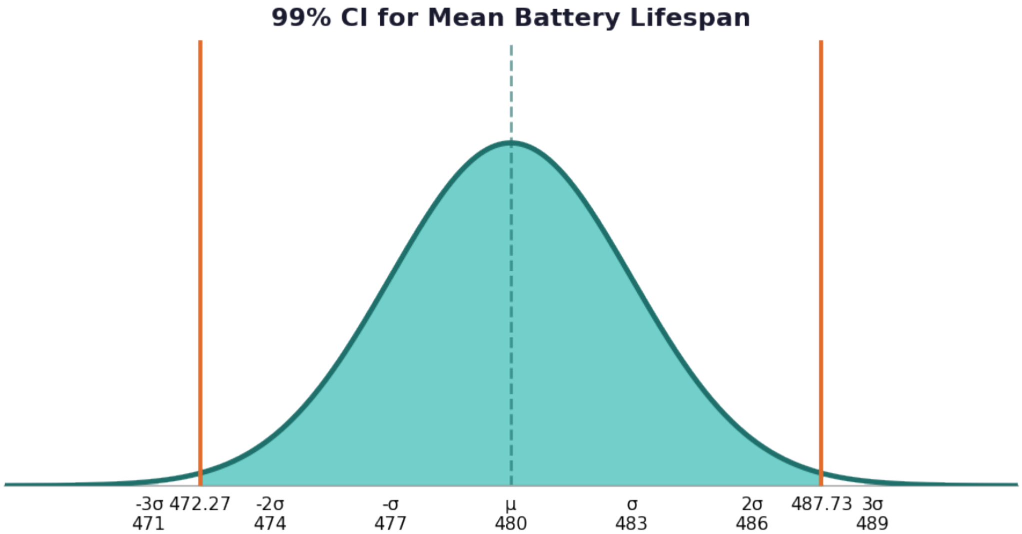
10. Answer: 99% CI: (472.27, 487.73). We are 99% confident the true mean battery lifespan is between 472.27 and 487.73 hours.



- Condition 1: The sample is a simple random sample — satisfied.
- Condition 2: The population is normally distributed — satisfied (stated in problem).
- Step 1: $SE = \sigma / \sqrt{n} = 45 / \sqrt{225} = 45 / 15 = 3$.
- Step 2: $ME = z^* \times SE = 2.576 \times 3 = 7.728 \approx 7.73$.
- Step 3: Lower bound = 480 – 7.73 = 472.27.
- Step 4: Upper bound = 480 + 7.73 = 487.73.
- Conclusion: We are 99% confident the true mean battery lifespan lies between 472.27 hours and 487.73 hours.

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