

# Confidence Intervals (t-Distribution)

Statistics Worksheet · Grade 11–College

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Learning Objectives

- Calculate the point estimate, margin of error, standard error, and sample standard deviation from a given confidence interval
- Determine the critical t-value using degrees of freedom and confidence level
- Write a statistically correct interpretation of a confidence interval in context

## Problems

1. A 95% confidence interval for the average lifespan of AA batteries is given as (430, 470) minutes. What is the point estimate (sample mean)?

$$\bar{x} = \frac{\text{Lower Limit} + \text{Upper Limit}}{2}$$

2. Using the confidence interval (430, 470) minutes, calculate the margin of error.

$$E = \text{Upper Limit} - \bar{x}$$

3. A confidence interval is reported as (118, 134). Find the point estimate and the margin of error.

$$\bar{x} = \frac{118 + 134}{2}, \quad E = \frac{134 - 118}{2}$$

4. Find the critical t-value ( $t^*$ ) for a 95% confidence interval with a sample size of 30. Use the formula: alpha divided by 2, where alpha equals 1 minus the confidence level.

$$\alpha/2 = \frac{1 - 0.95}{2} = 0.025, \quad df = n - 1 = 29$$

5. Using the AA battery example with a 95% confidence interval of (430, 470) minutes, a sample size of 30, and a critical t-value of 2.045, find the sample standard deviation.

$$E = t^* \cdot \frac{s}{\sqrt{n}} \Rightarrow s = \frac{E \cdot \sqrt{n}}{t^*}$$

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6. Using the AA battery example with a 95% confidence interval of (430, 470) minutes, a sample size of 30, and a critical t-value of 2.045, calculate the standard error.

$$SE = \frac{s}{\sqrt{n}}$$

7. A study reports a 90% confidence interval for average sleep duration as (6.4, 7.8) hours based on a sample of 25 people. Find the critical t-value for this interval.

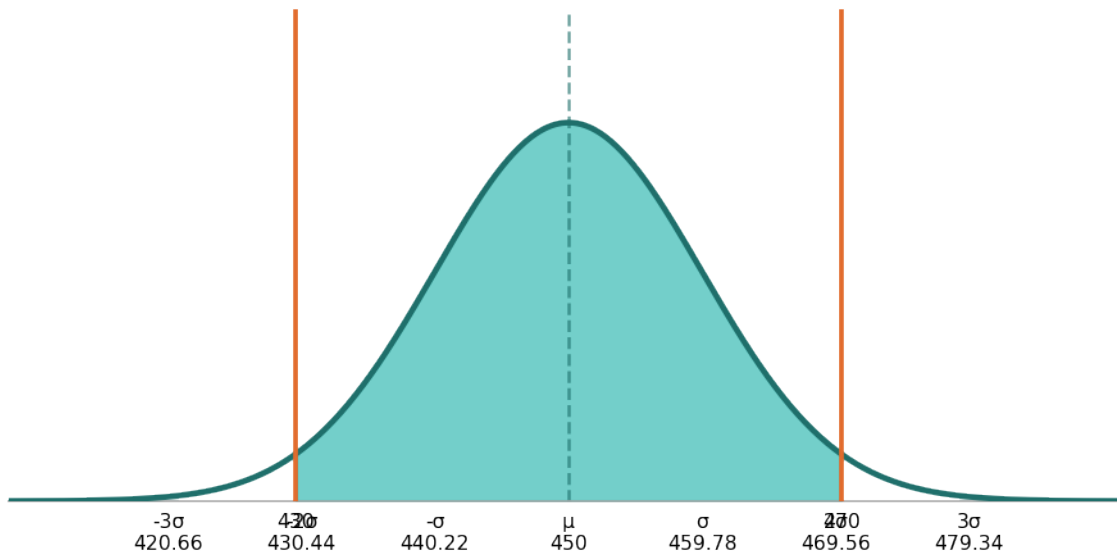
$$\alpha/2 = \frac{1 - 0.90}{2} = 0.05, \quad df = 24$$

8. A 90% confidence interval for average sleep duration is (6.4, 7.8) hours, based on a sample of 25 people. The critical t-value is 1.711. Find the sample standard deviation and the standard error.

$$E = \frac{7.8 - 6.4}{2}, \quad s = \frac{E \cdot \sqrt{n}}{t^*}, \quad SE = \frac{s}{\sqrt{n}}$$

9. Write a statistically correct interpretation of the following confidence interval: A 95% confidence interval for the average lifespan of AA batteries is (430, 470) minutes.

**95% Confidence Interval: (430, 470) minutes**



**10.** A researcher claims that the average monthly electricity bill in a city is between \$112 and \$148, based on a random sample of 40 households with a 99% confidence level. Find: (a) the point estimate, (b) the margin of error, (c) the critical t-value, (d) the standard error, and (e) the sample standard deviation. Then write a correct interpretation of the interval.

$$\bar{x} = \frac{112 + 148}{2}, \quad E = \frac{148 - 112}{2}, \quad t^* = t_{\alpha/2, df=39}, \quad s = \frac{E\sqrt{n}}{t^*}, \quad SE = \frac{s}{\sqrt{n}}$$


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# Confidence Intervals (t-Distribution) — Answer Key

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## Answer Key

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### 1. Answer: 450 minutes

- Point estimate = midpoint of the confidence interval
- $x_{\blacksquare} = (430 + 470) / 2 = 900 / 2 = 450$  minutes

### 2. Answer: 20 minutes

- Point estimate  $x_{\blacksquare} = 450$
- Margin of error  $E = 470 - 450 = 20$  minutes
- Alternatively,  $E = (470 - 430) / 2 = 40 / 2 = 20$  minutes

### 3. Answer: Point estimate = 126; Margin of error = 8

- $x_{\blacksquare} = (118 + 134) / 2 = 252 / 2 = 126$
- $E = (134 - 118) / 2 = 16 / 2 = 8$

### 4. Answer: $t^* \approx 2.045$

- $\alpha = 1 - 0.95 = 0.05$
- $\alpha/2 = 0.025$
- Degrees of freedom =  $n - 1 = 30 - 1 = 29$
- Using inverse-t:  $t^*(0.025, df=29) \approx 2.045$

### 5. Answer: $s \approx 53.57$ minutes

- $E = 20, t^* = 2.045, n = 30$
- $s = (E \times \sqrt{n}) / t^*$
- $s = (20 \times \sqrt{30}) / 2.045$
- $s = (20 \times 5.477) / 2.045 \approx 109.54 / 2.045 \approx 53.57$  minutes

### 6. Answer: $SE \approx 9.78$ minutes

- From Problem 5,  $s \approx 53.57$
- $SE = s / \sqrt{n} = 53.57 / \sqrt{30}$
- $SE = 53.57 / 5.477 \approx 9.78$  minutes
- Check:  $t^* \times SE = 2.045 \times 9.78 \approx 20 \checkmark$

### 7. Answer: $t^* \approx 1.711$

- $\alpha = 1 - 0.90 = 0.10$
- $\alpha/2 = 0.05$
- $df = n - 1 = 25 - 1 = 24$
- Using inverse-t:  $t^*(0.05, df=24) \approx 1.711$

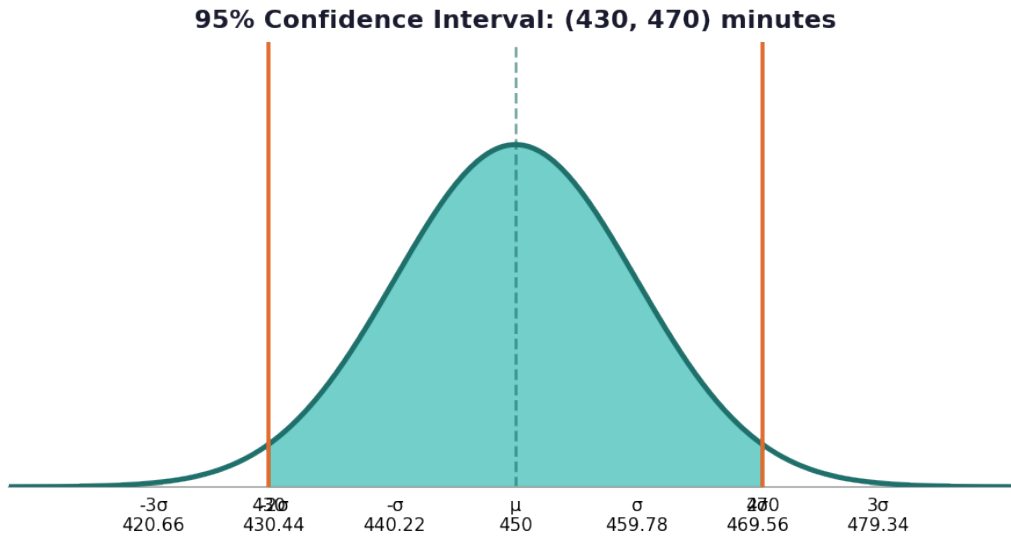
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**8. Answer: E = 0.7 hrs; s ≈ 2.047 hrs; SE ≈ 0.409 hrs**

- $x_{\square} = (6.4 + 7.8)/2 = 7.1$ ;  $E = 7.8 - 7.1 = 0.7$  hours
- $s = (E \times \sqrt{n}) / t^* = (0.7 \times \sqrt{25}) / 1.711 = (0.7 \times 5) / 1.711 = 3.5 / 1.711 \approx 2.047$  hours
- $SE = s / \sqrt{n} = 2.047 / 5 \approx 0.409$  hours
- Check:  $t^* \times SE = 1.711 \times 0.409 \approx 0.7$  ✓

**9. Answer: We are 95% confident that the true mean lifespan of all AA batteries falls between 430 and 470 minutes.**



- Identify the confidence level: 95%
- Identify the parameter: the TRUE POPULATION mean lifespan of AA batteries
- Identify the interval bounds: 430 to 470 minutes
- Correct interpretation: 'We are 95% confident that the true mean lifespan of AA batteries is between 430 and 470 minutes.' — This does NOT mean there is a 95% probability that a single battery lasts in this range.

**10. Answer:  $x_{\square} = \$130$ ; E = \$18;  $t^* \approx 2.708$ ; SE ≈ 6.65; s ≈ 42.04**

- (a)  $x_{\square} = (112 + 148) / 2 = 260 / 2 = 130$
- (b)  $E = 148 - 130 = 18$
- (c)  $\alpha = 1 - 0.99 = 0.01$ ;  $\alpha/2 = 0.005$ ;  $df = 40 - 1 = 39$ ;  $t^*(0.005, df=39) \approx 2.708$
- (d)  $SE = E / t^* = 18 / 2.708 \approx 6.65$
- (e)  $s = SE \times \sqrt{n} = 6.65 \times \sqrt{40} = 6.65 \times 6.325 \approx 42.04$
- Interpretation: We are 99% confident that the true mean monthly electricity bill for all households in the city is between \$112 and \$148.

