Confidence Intervals Using the Paired t-Design

Statistics Worksheet · Grade 11–12 / AP Statistics

Name: ________________________________ Date: ____________________

Learning Objectives

- Identify when a matched-pair (paired t) design is appropriate for two samples
- Compute the mean difference, standard deviation of differences, and degrees of freedom for a paired data set
- Construct and interpret a confidence interval for the population mean difference using the paired t formula

Problems

1. A researcher records the weight (in pounds) of 5 students before and after a 4-week diet program. The differences (Before minus After) are: 3, 5, 2, 4, 6. What is the mean difference?

$$\bar{d} = \frac{\sum d_i}{n}$$

2. Explain why the paired t-design is also called the 'matched pair design.' Give one real-world example where this design is appropriate.

3. A paired study compares two corn seed types (Regular vs. Kiln-Dried) on the same plots. The differences (Regular minus Kiln-Dried) for 6 plots are shown below. Complete the table by calculating each difference.

Plot	Regular (lb/acre)	Kiln-Dried (lb/acre)	Difference (d)
1	1903	2009	
2	1935	1915	
3	1910	2011	
4	2496	2180	
5	2108	1990	
6	1511	1535	

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4. Using the six differences from Problem 3 (-106, 20, -101, 316, 118, -24), find the mean difference and the degrees of freedom for the paired t procedure.

$$\bar{d} = \frac{\sum d_i}{n}, \quad df = n - 1$$

5. The differences from a paired study of 11 corn plots have a mean difference of -33.727 and a standard deviation of differences of 66.171. Write the formula for the paired t confidence interval and identify what each variable represents.

$$\bar{d} \pm t^* \cdot \frac{s_d}{\sqrt{n}}$$

6. A paired t study has 11 pairs of observations. The standard deviation of differences is 66.171. Find the standard error of the mean difference.

$$SE_{\bar{d}} = \frac{s_d}{\sqrt{n}}$$

7. Using the paired t distribution with 10 degrees of freedom at the 95% confidence level, the critical value t* is 2.228. The mean difference is -33.727 and the standard error is 19.953. Construct the 95% confidence interval for the population mean difference.

$$\bar{d} \pm t^* \cdot \frac{s_d}{\sqrt{n}}$$

8. Based on the 95% confidence interval for the mean difference of regular versus kiln-dried corn yields found in Problem 7, can you conclude that there is a statistically significant difference between the two seed types? Explain your reasoning.

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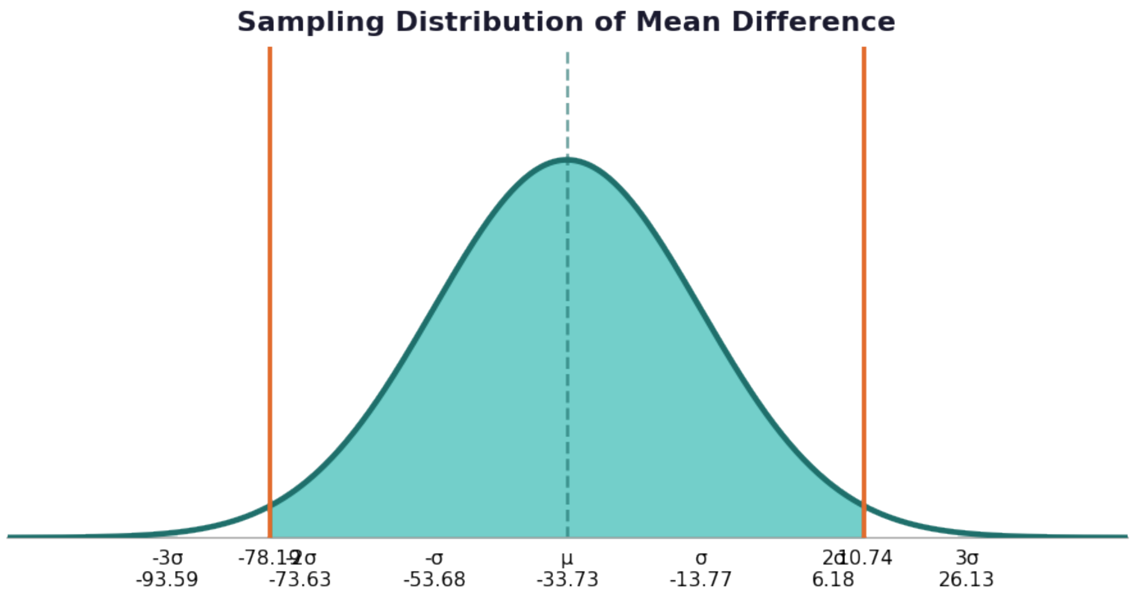

9. A physical therapist measures the range of motion (in degrees) of 8 patients' shoulders before and after treatment. The differences (After minus Before) are: 12, 9, 15, 7, 10, 13, 8, 6. The standard deviation of differences is 2.97. Construct a 95% confidence interval for the population mean improvement. Use t* = 2.365 for df = 7.

$$\bar{d} \pm t^* \cdot \frac{s_d}{\sqrt{n}}$$

10. A study tests whether a new fertilizer increases tomato yield (pounds per plant). Eight matched plots are used; each plot is split, with one half receiving the new fertilizer and the other half receiving the standard fertilizer. The data are shown below. At the 99% confidence level (use t* = 3.499, df = 7), construct the confidence interval for the mean difference (New minus Standard) and state whether the new fertilizer significantly increases yield.

Plot	New Fertilizer	Standard Fertilizer	Difference (d)	d^2
1	12.1	10.5	1.6	2.56
2	11.4	9.8	1.6	2.56
3	13.2	11.0	2.2	4.84
4	10.9	10.1	0.8	0.64
5	12.7	11.5	1.2	1.44

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Plot	New Fertilizer	Standard Fertilizer	Difference (d)	d^2
6	14.0	12.3	1.7	2.89
7	11.8	10.2	1.6	2.56
8	13.5	11.2	2.3	5.29

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Confidence Intervals Using the Paired t-Design — Answer Key

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Answer Key

1. Answer: Mean difference = 4

- List the differences: 3, 5, 2, 4, 6
- Sum of differences = 3 + 5 + 2 + 4 + 6 = 20
- n = 5
- Mean difference = 20 / 5 = 4

2. Answer: Each observation in Sample 1 is paired with a specific observation in Sample 2 (e.g., same subject measured twice). Example: measuring blood pressure before and after medication on the same patients.

- In a matched pair design, pairs share a natural link (same individual, same plot of land, etc.).
- This controls for individual variability and focuses the analysis on the difference within each pair.
- Real-world example: testing the same students before and after a tutoring program.

3. Answer: Differences: -106, 20, -101, 316, 118, -24

Plot	Regular (lb/acre)	Kiln-Dried (lb/acre)	Difference (d)
1	1903	2009	-106
2	1935	1915	20
3	1910	2011	-101
4	2496	2180	316
5	2108	1990	118
6	1511	1535	-24

- Subtract Kiln-Dried from Regular for each plot.
- Plot 1: 1903 - 2009 = -106
- Plot 2: 1935 - 1915 = 20
- Plot 3: 1910 - 2011 = -101
- Plot 4: 2496 - 2180 = 316
- Plot 5: 2108 - 1990 = 118
- Plot 6: 1511 - 1535 = -24

4. Answer: Mean difference = 37.17, df = 5

- Sum of differences = -106 + 20 + (-101) + 316 + 118 + (-24) = 223
- n = 6

- Mean difference = 223 / 6 ≈ 37.17
- Degrees of freedom = n - 1 = 6 - 1 = 5

5. Answer: d-bar is the mean of the differences; t* is the critical value from the t-distribution; s_d is the standard deviation of the differences; n is the number of pairs.

- The formula is: d-bar ± t* × (s_d / sqrt(n))
- d-bar = -33.727 (mean of the paired differences)
- t* = critical value from the t-table at the chosen confidence level with df = n - 1
- s_d = 66.171 (standard deviation of the paired differences)
- n = 11 (number of matched pairs)

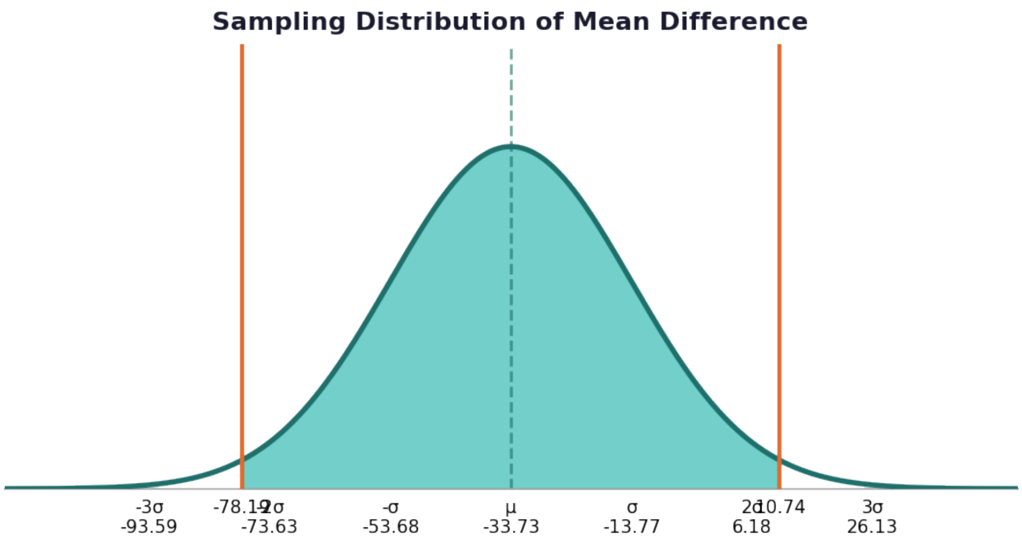
6. Answer: SE ≈ 19.953

- SE = s_d / sqrt(n) = 66.171 / sqrt(11)
- sqrt(11) ≈ 3.3166
- SE = 66.171 / 3.3166 ≈ 19.953

7. Answer: 95% CI: (-78.19, 10.74)

- Margin of error = t* × SE = 2.228 × 19.953 ≈ 44.46
- Lower bound = -33.727 - 44.46 ≈ -78.19
- Upper bound = -33.727 + 44.46 ≈ 10.74
- 95% Confidence Interval: (-78.19, 10.74)

8. Answer: No significant difference; zero is contained in the interval (-78.19, 10.74), so we cannot conclude the population mean difference is nonzero at the 95% confidence level.



- The 95% CI is (-78.19, 10.74).
- Since 0 is within this interval, it is plausible that the true mean difference is 0.
- Therefore, there is not enough evidence at the 95% confidence level to conclude a significant difference between regular and kiln-dried corn yields.

9. Answer: 95% CI: (7.518, 12.482)

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- Sum of differences = 12 + 9 + 15 + 7 + 10 + 13 + 8 + 6 = 80
- n = 8, so mean difference = 80 / 8 = 10
- SE = s_d / sqrt(n) = 2.97 / sqrt(8) = 2.97 / 2.8284 ≈ 1.050
- Margin of error = t* × SE = 2.365 × 1.050 ≈ 2.483
- Lower bound = 10 - 2.483 ≈ 7.518
- Upper bound = 10 + 2.483 ≈ 12.482
- 95% CI: (7.518, 12.482) — since 0 is not in the interval, the treatment shows a significant improvement.

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10. Answer: Mean diff ≈ 1.625; s_d ≈ 0.492; 99% CI: (0.993, 2.257) — Yes, significant increase.

- Differences: 1.6, 1.6, 2.2, 0.8, 1.2, 1.7, 1.6, 2.3
- Sum of differences = 13.0; n = 8; mean difference = 13.0 / 8 = 1.625
- Sum of d^2 = 2.56+2.56+4.84+0.64+1.44+2.89+2.56+5.29 = 22.78
- s_d^2 = [22.78 - 8×$(1.625)^2$] / (8-1) = [22.78 - 21.125] / 7 = 1.655 / 7 ≈ 0.2364
- s_d = sqrt(0.2364) ≈ 0.4862
- SE = s_d / sqrt(8) = 0.4862 / 2.8284 ≈ 0.1719
- Margin of error = t* × SE = 3.499 × 0.1719 ≈ 0.602
- Lower bound = 1.625 - 0.602 ≈ 1.023; Upper bound = 1.625 + 0.602 ≈ 2.227
- 99% CI: approximately (1.023, 2.227). Since 0 is NOT in the interval, the new fertilizer produces a significantly higher yield at the 99% confidence level.

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