Geometric Probability Distribution

Statistics Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Identify the conditions of a geometric experiment and distinguish it from a binomial experiment
- Apply the geometric probability formula P(X = x) = p · q^(x–1) to solve probability problems
- Calculate the mean, variance, and standard deviation of a geometric distribution

Problems

1. A fair coin is flipped until a tail appears. Which of the following describes why this is a geometric experiment and NOT a binomial experiment? Select the correct reason.

A) There are more than 2 outcomes B) There is no fixed number of trials C) Trials are dependent D) The probability changes each flip

2. A standard six-sided die is rolled until a 3 appears. Identify the values of p, q, and what X represents in this geometric experiment.

$$p = ?, \quad q = ?, \quad X = ?$$

3. A free-throw shooter makes 70% of their shots. Find the probability that the first successful free throw occurs on the second attempt.

$$P(X = 2) = p \cdot q^{x-1}$$

4. Studies show that 5% of light bulbs produced in a factory are defective. What is the probability that the first defective bulb is found on the third inspection?

$$P(X = 3) = p \cdot q^{x-1}$$

5. A salesperson closes a deal with 20% of all client calls. Find the mean (expected value) of the geometric distribution and interpret what it means in context.

$$\mu = \frac{1}{p}$$

Scan to watch

6. For a geometric distribution where the probability of success is 0.25, find the variance and standard deviation.

$$\sigma^2 = \frac{q}{p^2}, \quad \sigma = \sqrt{\frac{q}{p^2}}$$

7. According to a survey, 30% of adults in a city use public transportation daily. If adults are selected at random one at a time, find the probability that the first adult who uses public transportation daily is the fourth person selected.

$$P(X = 4) = p \cdot q^{x-1}$$

8. Refer to the table below. A geometric experiment has probability of success p = 0.40. Fill in the missing probability values P(X = x) for x = 1, 2, 3, 4, and 5. Round to four decimal places.

x	P(X = x)
1	
2	
3	
4	
5	

9. A quality control inspector checks items off an assembly line. Each item has a 4% chance of being defective. Find the probability that the FIRST defective item is found on or before the third inspection — that is, find P(X is less than or equal to 3). Round to four decimal places.

$$P(X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3)$$

10. A multiple-choice test has 5 answer choices per question, only one of which is correct. A student randomly guesses on every question. Using the geometric distribution, find: (a) the probability that the student gets the FIRST correct answer on the 6th question, (b) the mean number of questions until the first correct guess, and (c) the standard deviation. Round all answers to four decimal places.

Scan to watch

$p = \frac{1}{5}, \quad q = \frac{4}{5}$

Scan to watch

Geometric Probability Distribution — Answer Key

Statistics Worksheet · Grade 10–12

Answer Key

1. Answer: B) There is no fixed number of trials

- A geometric experiment shares all binomial conditions EXCEPT condition #1.
- The key difference is that a geometric experiment has NO fixed number of trials — you keep going until the first success.
- All other conditions hold: 2 outcomes (H or T), independent trials, constant probability (p = 0.5).
- Answer: B

2. Answer: p = 1/6, q = 5/6, X = the roll on which the first 3 appears

- Success = rolling a 3. There is 1 favorable outcome out of 6 total.
- p = 1/6 ≈ 0.1667
- q = 1 – p = 5/6 ≈ 0.8333
- X = the trial number on which the first 3 is rolled (no fixed maximum).

3. Answer: P(X = 2) = 0.21

- Identify: p = 0.70, q = 1 – 0.70 = 0.30, X = 2.
- Apply the formula: P(X = 2) = p · q^(2–1) = 0.70 · 0.30^1.
- P(X = 2) = 0.70 × 0.30 = 0.21.
- There is a 21% chance the first success occurs on the second attempt.

4. Answer: P(X = 3) ≈ 0.0451

- Identify: p = 0.05, q = 1 – 0.05 = 0.95, X = 3.
- Apply the formula: P(X = 3) = 0.05 · 0.95^(3–1) = 0.05 · 0.95^2.
- 0.95^2 = 0.9025.
- P(X = 3) = 0.05 × 0.9025 = 0.045125 ≈ 0.0451 or about 4.51%.

5. Answer: μ = 5 calls

- Identify: p = 0.20.
- Apply the mean formula: μ = 1/p = 1/0.20.
- μ = 5.
- Interpretation: On average, the salesperson must make 5 calls before closing a deal.

6. Answer: Variance = 12, Standard Deviation ≈ 3.46

- Identify: p = 0.25, q = 1 – 0.25 = 0.75.
- Variance: $\sigma^2 = q / p^2 = 0.75 / (0.25)^2 = 0.75 / 0.0625 = 12$.
- Standard Deviation: $\sigma = \sqrt{12} = \sqrt{(4 \times 3)} = 2\sqrt{3} \approx 3.464$.
- Variance = 12, Standard Deviation ≈ 3.46.

7. Answer: P(X = 4) ≈ 0.1029

- Identify: p = 0.30, q = 0.70, X = 4.

Scan to watch

- Apply the formula: P(X = 4) = 0.30 · 0.70^(4–1) = 0.30 · 0.70^3.
- 0.70^3 = 0.343.
- P(X = 4) = 0.30 × 0.343 = 0.1029 or about 10.29%.

8. Answer: See completed table

x	P(X = x)
1	0.4000
2	0.2400
3	0.1440
4	0.0864
5	0.0518

- Use P(X = x) = p · q^(x–1) with p = 0.40 and q = 0.60.
- P(X=1) = 0.40 · 0.60^0 = 0.40 · 1 = 0.4000
- P(X=2) = 0.40 · 0.60^1 = 0.40 · 0.60 = 0.2400
- P(X=3) = 0.40 · 0.60^2 = 0.40 · 0.36 = 0.1440
- P(X=4) = 0.40 · 0.60^3 = 0.40 · 0.216 = 0.0864
- P(X=5) = 0.40 · 0.60^4 = 0.40 · 0.1296 = 0.0518

9. Answer: P(X ≤ 3) ≈ 0.1153

- Identify: p = 0.04, q = 0.96.
- P(X=1) = 0.04 · 0.96^0 = 0.04 · 1 = 0.0400
- P(X=2) = 0.04 · 0.96^1 = 0.04 · 0.96 = 0.0384
- P(X=3) = 0.04 · 0.96^2 = 0.04 · 0.9216 = 0.036864 ≈ 0.0369
- P(X ≤ 3) = 0.0400 + 0.0384 + 0.0369 = 0.1153
- There is about an 11.53% chance the first defective item appears within the first three inspections.

10. Answer: (a) P(X=6) ≈ 0.0655, (b) μ = 5 questions, (c) σ ≈ 4.4721

- Identify: p = 1/5 = 0.20, q = 4/5 = 0.80.
- (a) P(X=6) = p · q^(6–1) = 0.20 · 0.80^5 = 0.20 · 0.32768 = 0.065536 ≈ 0.0655.
- (b) Mean: μ = 1/p = 1/0.20 = 5. On average, 5 questions must be answered before the first correct guess.
- (c) Variance: $\sigma^2 = q/p^2 = 0.80/(0.20)^2 = 0.80/0.04 = 20$.
- Standard deviation: $\sigma = \sqrt{20} = 2\sqrt{5} \approx 4.4721$.

Scan to watch