Hypothesis Testing for One Sample Proportion

Statistics Worksheet · Grade 11–College

Name: _____ Date: _____

Learning Objectives

- Write null and alternative hypotheses for one-sample proportion tests
- Verify normality and independence conditions before conducting a test
- Calculate the z-test statistic for a proportion and interpret the p-value to draw a conclusion

Problems

1. A survey claims that 60% of adults drink coffee daily. Identify the null and alternative hypotheses to test whether the true proportion is different from 60%. State which tail type this test is.

H_0: $p = 0.60$

H_1: $p \neq 0.60$

2. A school administrator believes fewer than 70% of students pass the state math exam. Write the null and alternative hypotheses and identify the tail type.

H_0: $p = 0.70$

H_1: $p < 0.70$

3. Before conducting a hypothesis test for a proportion, you must verify the normality condition. A sample of n = 50 is taken from a population where the claimed proportion is p = 0.30. Check whether the normality condition is satisfied using the Rule of Thumb: n times p greater than 10 and n times q greater than 10.

$n \cdot p > 10 \quad \text{and} \quad n \cdot q > 10$

4. A quality control manager claims that more than 85% of products pass inspection. In a random sample of 80 products, 71 passed. Verify the normality condition using the claimed

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proportion, then identify the tail type of the test.

H_0: p = 0.85

H_1: p > 0.85

$$n \cdot p > 10 \quad \text{and} \quad n \cdot q > 10$$

5. In a random sample of 100 voters, 55 support a ballot measure. The population proportion supporting it is claimed to be 50%. Calculate the sample proportion p-hat and the z-test statistic.

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

6. A health organization claims that 30% of teenagers smoke. A researcher suspects the proportion is lower. In a random sample of 200 teenagers, 48 are found to smoke. At a significance level of 5%, compute the z-test statistic and state whether you reject or fail to reject the null hypothesis.

H_0: p = 0.30

H_1: p < 0.30

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

7. Ryan, a basketball player, made 40% of his free throws last season. After training over the summer, he made 25 out of 40 free throw attempts in the first eight games of the new season. Test at a 1% significance level whether his free throw proportion has improved. Show all four steps: hypotheses, conditions, test statistic, and conclusion.

H_0: p = 0.40

H_1: p > 0.40

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

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8. A pharmaceutical company claims that 75% of patients respond positively to a new drug. In a clinical trial of 150 patients, 104 responded positively. At a 5% significance level, conduct a two-tailed hypothesis test to determine whether the true response proportion differs from 75%. Include all four steps and a conclusion.

H_0: $p = 0.75$

H_1: $p \neq 0.75$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

9. An election poll taken before an election showed that 52% of 400 randomly selected voters planned to vote for Candidate A. At a 1% significance level, test whether the proportion of voters planning to vote for Candidate A is significantly greater than 50%. Compute the p-value (using the z-score) and compare it to the significance level to make your decision.

H_0: $p = 0.50$

H_1: $p > 0.50$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

10. A consumer advocacy group suspects that the proportion of defective electronics from a manufacturer is higher than the advertised rate of 2%. They randomly test 500 units and find 18 defective. At a 5% significance level, perform a complete hypothesis test. Verify all conditions, compute the z-statistic and p-value, and write a formal conclusion in context.

H_0: $p = 0.02$

H_1: $p > 0.02$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

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Hypothesis Testing for One Sample Proportion — Answer Key

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Answer Key

1. Answer: H_0: $p = 0.60$, H_1: $p \neq 0.60$; Two-tailed test

- The claim is that the proportion equals 60%, so H_0: $p = 0.60$.
- We are testing whether the proportion is *different* (not specifically higher or lower), so H_1: $p \neq 0.60$.
- Because H_1 uses $\neq$, this is a two-tailed test.

2. Answer: H_0: $p = 0.70$, H_1: $p < 0.70$; Left-tailed test

- The administrator suspects the proportion is less than 70%, so H_1: $p < 0.70$.
- The null hypothesis always uses equality: H_0: $p = 0.70$.
- Because H_1 uses $<$, this is a left-tailed test.

3. Answer: $np = 15 > 10$ and $nq = 35 > 10$; Normality condition is satisfied

- Given: $n = 50$, $p = 0.30$, so $q = 1 - 0.30 = 0.70$.
- Check $n \cdot p$: $50 \times 0.30 = 15 > 10$. ✓
- Check $n \cdot q$: $50 \times 0.70 = 35 > 10$. ✓
- Both conditions are met, so the normality condition is satisfied.

4. Answer: $np = 68 > 10$ and $nq = 12 > 10$; Normality satisfied. Right-tailed test.

- Given: $n = 80$, $p_0 = 0.85$, $q = 0.15$.
- $n \cdot p = 80 \times 0.85 = 68 > 10$. ✓
- $n \cdot q = 80 \times 0.15 = 12 > 10$. ✓
- Normality condition is satisfied.
- Since the manager claims *more than* 85%, H_1: $p > 0.85$ → right-tailed test.

5. Answer: $\hat{p} = 0.55$; $z \approx 1.00$

- $\hat{p} = 55/100 = 0.55$
- $p_0 = 0.50$, $q_0 = 0.50$, $n = 100$
- Standard error $= \sqrt{(0.50 \times 0.50 / 100)} = \sqrt{(0.0025)} = 0.05$
- $z = (0.55 - 0.50) / 0.05 = 0.05 / 0.05 = 1.00$
- The z-test statistic is $z = 1.00$.

6. Answer: $\hat{p} = 0.24$; $z \approx -1.85$; Reject H_0 at $\alpha = 0.05$

- $\hat{p} = 48/200 = 0.24$
- $p_0 = 0.30$, $q_0 = 0.70$, $n = 200$
- Standard error $= \sqrt{(0.30 \times 0.70 / 200)} = \sqrt{(0.00105)} \approx 0.0324$
- $z = (0.24 - 0.30) / 0.0324 \approx -0.06 / 0.0324 \approx -1.85$
- Left-tailed critical value at $\alpha = 0.05$: $z_\alpha = -1.645$

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• Since $z = -1.85 < -1.645$, we reject H_0.
• Conclusion: There is sufficient evidence that the proportion of teenagers who smoke is less than 30%.

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7. Answer: $\hat{p} = 0.625$; $z \approx 2.90$; Reject H_0 at $\alpha = 0.01$

• Step 1 — Hypotheses: H_0: $p = 0.40$, H_1: $p > 0.40$ (right-tailed)
• Step 2 — Conditions: $n \cdot p = 40 \times 0.40 = 16 > 10$ ✓; $n \cdot q = 40 \times 0.60 = 24 > 10$ ✓; sample is random ✓
• Step 3 — Test Statistic: $\hat{p} = 25/40 = 0.625$; SE $= \sqrt{(0.40 \times 0.60/40)} = \sqrt{(0.006)} \approx 0.0775$; $z = (0.625 - 0.40)/0.0775 \approx 2.90$
• Step 4 — Conclusion: Critical value at $\alpha = 0.01$ right-tailed: $z = 2.326$. Since $2.90 > 2.326$, reject H_0.
• There is sufficient evidence at the 1% significance level that Ryan's free throw proportion has improved.

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8. Answer: $\hat{p} \approx 0.693$; $z \approx -1.62$; Fail to reject H_0 at $\alpha = 0.05$

• Step 1 — Hypotheses: H_0: $p = 0.75$, H_1: $p \neq 0.75$ (two-tailed)
• Step 2 — Conditions: $n \cdot p = 150 \times 0.75 = 112.5 > 10$ ✓; $n \cdot q = 150 \times 0.25 = 37.5 > 10$ ✓; random sample ✓
• Step 3 — Test Statistic: $\hat{p} = 104/150 \approx 0.6933$; SE $= \sqrt{(0.75 \times 0.25/150)} = \sqrt{(0.00125)} \approx 0.0354$; $z = (0.6933 - 0.75)/0.0354 \approx -1.60$
• Step 4 — Critical values for two-tailed at $\alpha = 0.05$: $z = \pm 1.96$. Since $-1.96 < -1.60 < 1.96$, fail to reject H_0.
• Conclusion: There is not sufficient evidence at the 5% significance level that the response proportion differs from 75%.

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9. Answer: $z = 0.80$; p-value ≈ 0.2119; Fail to reject H_0

• Step 1 — Hypotheses: H_0: $p = 0.50$, H_1: $p > 0.50$ (right-tailed)
• Step 2 — Conditions: $n \cdot p = 400 \times 0.50 = 200 > 10$ ✓; $n \cdot q = 200 > 10$ ✓; random sample ✓
• Step 3 — Test Statistic: $\hat{p} = 0.52$; SE $= \sqrt{(0.50 \times 0.50/400)} = \sqrt{(0.000625)} = 0.025$; $z = (0.52 - 0.50)/0.025 = 0.80$
• Step 4 — p-value: $P(Z > 0.80) = 1 - 0.7881 = 0.2119$. Since $0.2119 > 0.01$ (α), fail to reject H_0.
• Conclusion: There is not sufficient evidence at the 1% significance level that more than 50% of voters support Candidate A.

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10. Answer: $\hat{p} = 0.036$; $z \approx 2.55$; p-value ≈ 0.0054; Reject H_0 at $\alpha = 0.05$

• Step 1 — Hypotheses: H_0: $p = 0.02$, H_1: $p > 0.02$ (right-tailed)
• Step 2 — Conditions: $n \cdot p = 500 \times 0.02 = 10$, which is borderline but acceptable (≥ 10) ✓; $n \cdot q = 500 \times 0.98 = 490 > 10$ ✓; random sample ✓; population of all units is far larger than $10 \times 500 = 5000$ ✓
• Step 3 — Test Statistic: $\hat{p} = 18/500 = 0.036$; SE $= \sqrt{(0.02 \times 0.98/500)} = \sqrt{(0.0000392)} \approx 0.00626$; $z = (0.036 - 0.02)/0.00626 \approx 0.016/0.00626 \approx 2.55$
• Step 4 — p-value: $P(Z > 2.55) \approx 1 - 0.9946 = 0.0054$. Since $0.0054 < 0.05$ (α), reject H_0.
• Conclusion: There is sufficient evidence at the 5% significance level that the proportion of defective electronics exceeds the advertised rate of 2%. The manufacturer's claim appears to be understated.

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