Hypothesis Testing for Population Mean

Statistics Worksheet · Grade 11–College

Name: ______________________________ Date: _________________

Learning Objectives

- Write null and alternative hypotheses for population mean problems
- Calculate the z-test or t-test statistic and find the p-value
- Make a decision and write a conclusion in context using the significance level

Problems

1. A school claims the mean score on a reading test is 75 points. State the null and alternative hypotheses to test whether the true mean is different from 75.

H_0: $\mu = 75$

H_1: $\mu \neq 75$

2. A nutritionist believes the average daily sugar intake for teenagers is more than 80 grams. Write the null and alternative hypotheses for this claim and identify the tail of the test.

H_0: $\mu = 80$

H_1: $\mu > 80$

3. State whether each condition for hypothesis testing about a population mean is satisfied given: the sample of 40 students was randomly selected, the population of test scores is approximately normally distributed, and each student's score is independent of the others.

Condition	Requirement	Satisfied?
Random Sample	Sample must be randomly selected	Yes
Normality	Population is normally distributed OR $n \geq 30$	Yes
Independence	Each observation is independent	Yes

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4. A bottling company states the mean volume of soda in its cans is 355 mL with a known population standard deviation of 6 mL. A quality inspector takes a random sample of 36 cans and finds a sample mean of 352 mL. Calculate the z-test statistic.

H_0: μ = 355

H_1: μ ≠ 355

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

5. Using the z-test statistic z = −3.00 from Problem 4 and a significance level of α = 0.05 for a two-tailed test, find the p-value and state whether you reject or fail to reject the null hypothesis.

H_0: μ = 355

H_1: μ ≠ 355

$$p\text{-value} = 2 \cdot P(Z < -3.00)$$

6. The Survey of Study Habits and Attitudes (SSHA) has a known mean of 115 and standard deviation of 30 for U.S. college students. Monica randomly selects 25 students aged 30 or older and finds a sample mean of 118.6. At α = 0.05, test whether older students have better attitudes toward school. Calculate the z-statistic and find the p-value.

H_0: μ = 115

H_1: μ > 115

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

7. A manufacturer claims its light bulbs last an average of 1200 hours. A consumer group tests 49 randomly selected bulbs and finds a mean of 1185 hours with a known population standard deviation of 70 hours. At α = 0.01, test whether the true mean life is less than 1200 hours. Show all five steps.

H_0: μ = 1200

H_1: μ < 1200

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$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

8. A random sample of 16 college students has a mean GPA of 3.10 with a sample standard deviation of 0.40. The national mean GPA is claimed to be 3.00. Assuming GPA is approximately normally distributed and σ is unknown, use a t-test at α = 0.05 to test whether the true mean GPA is greater than 3.00. Calculate the t-statistic and compare to the critical value.

H_0: μ = 3.00

H_1: μ > 3.00

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

9. A city health department claims the mean blood pressure of adult males is 120 mmHg. A random sample of 64 adult males shows a mean of 124 mmHg with a known population standard deviation of 16 mmHg. At α = 0.05, perform a two-tailed hypothesis test. Find the z-statistic, p-value, and write a conclusion. Also identify the shaded rejection region on a standard normal curve.

H_0: μ = 120

H_1: μ ≠ 120

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

10. A pharmaceutical company claims its new drug reduces mean recovery time to 5 days. A clinical trial randomly assigns the drug to 36 patients. The sample mean recovery time is 5.8 days and the sample standard deviation is 2.4 days. The population standard deviation is unknown. At α = 0.01, use a t-test to determine whether the mean recovery time is greater than 5 days. Verify all conditions, compute the t-statistic, find the p-value range using t-distribution critical values, state your decision, and write a full conclusion in context.

H_0: μ = 5

H_1: μ > 5

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$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Hypothesis Testing for Population Mean — Answer Key

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Answer Key

1. Answer: $H_0: \mu = 75$; $H_1: \mu \neq 75$ (two-tailed)

- The claim is that the mean score is 75, so $\mu_0 = 75$.
- The null hypothesis always contains the equality: $H_0: \mu = 75$.
- Since we are testing whether it is 'different from' 75, this is two-tailed: $H_1: \mu \neq 75$.

2. Answer: $H_0: \mu = 80$; $H_1: \mu > 80$ (right-tailed)

- The claim is that the mean is MORE than 80 grams, so $\mu_0 = 80$.
- $H_0: \mu = 80$ (equality in the null hypothesis).
- $H_1: \mu > 80$ — this is a right-tailed test because we are looking for evidence of a greater mean.

3. Answer: All three conditions are satisfied; proceed without caution.

- Condition 1 (Random): The sample was randomly selected — satisfied.
- Condition 2 (Normality): The population is approximately normal — satisfied.
- Condition 3 (Independence): Each score is independent — satisfied.
- All conditions met, so the hypothesis test is valid.

4. Answer: $z = -3.00$

- Identify values: $\bar{x} = 352$, $\mu_0 = 355$, $\sigma = 6$, $n = 36$.
- Standard error $= \sigma / \sqrt{n} = 6 / \sqrt{36} = 6 / 6 = 1$.
- $z = (352 - 355) / 1 = -3 / 1 = -3.00$.
- The test statistic is $z = -3.00$.

5. Answer: p-value ≈ 0.0026; Reject H_0

$$\text{p-value} = 2 \times 0.0013 = 0.0026$$

$$0.0026 < 0.05 \rightarrow \text{Reject } H_0$$

- $z = -3.00$ for a two-tailed test.
- $P(Z < -3.00) \approx 0.0013$ from the standard normal table.
- p-value $= 2 \times 0.0013 = 0.0026$.
- Since p-value $= 0.0026 < \alpha = 0.05$, reject H_0.
- Conclusion: There is sufficient evidence that the mean volume is not 355 mL.

6. Answer: $z \approx 0.60$; p-value ≈ 0.2743; Fail to reject H_0

- Given: $\bar{x} = 118.6$, $\mu_0 = 115$, $\sigma = 30$, $n = 25$.
- Standard error $= 30 / \sqrt{25} = 30 / 5 = 6$.
- $z = (118.6 - 115) / 6 = 3.6 / 6 = 0.60$.

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- For a right-tailed test, p-value = $P(Z > 0.60) = 1 - 0.7257 = 0.2743$.
- Since $0.2743 > 0.05$, we fail to reject H_0.
- Conclusion: There is not sufficient evidence that older students have better attitudes.

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7. Answer: $z = -1.50$; p-value ≈ 0.0668; Fail to reject H_0

- Step 1 — Hypotheses: H_0: $\mu = 1200$, H_1: $\mu < 1200$ (left-tailed).
- Step 2 — Conditions: Random sample ✓, normal population ($n = 49 \geq 30$) ✓, independent ✓.
- Step 3 — Test statistic: $z = (1185 - 1200) / (70/\sqrt{49}) = -15 / 10 = -1.50$.
- Step 4 — p-value: $P(Z < -1.50) \approx 0.0668$.
- Step 5 — Decision: $0.0668 > 0.01$, so fail to reject H_0.
- Conclusion: There is insufficient evidence at $\alpha = 0.01$ that the mean life is less than 1200 hours.

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8. Answer: $t = 1.00$; critical value $t = 1.753$; Fail to reject H_0

- Given: $\bar{x} = 3.10$, $\mu_0 = 3.00$, $s = 0.40$, $n = 16$.
- Degrees of freedom: $df = n - 1 = 15$.
- $t = (3.10 - 3.00) / (0.40/\sqrt{16}) = 0.10 / 0.10 = 1.00$.
- Critical value for right-tailed t-test: $t(0.05, 15) = 1.753$.
- Since $t = 1.00 < 1.753$, fail to reject H_0.
- Conclusion: There is not sufficient evidence that the mean GPA exceeds 3.00.

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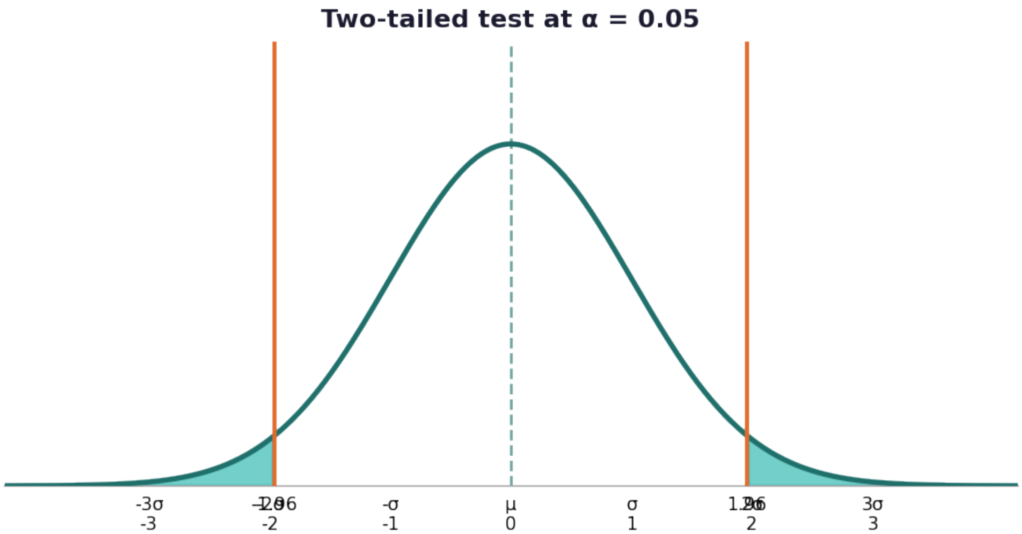
9. Answer: $z = 2.00$; p-value ≈ 0.0456; Reject H_0



- Step 1: H_0: $\mu = 120$, H_1: $\mu \neq 120$ (two-tailed).
- Step 2: Conditions met — random sample, $n = 64 \geq 30$, independent.
- Step 3: $z = (124 - 120) / (16/\sqrt{64}) = 4 / 2 = 2.00$.
- Step 4: p-value = $2 \times P(Z > 2.00) = 2 \times 0.0228 = 0.0456$.
- Step 5: $0.0456 < 0.05$, so reject H_0.
- Conclusion: There is sufficient evidence at $\alpha = 0.05$ that the mean blood pressure differs from 120 mmHg.

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10. Answer: $t = 2.00$; $0.025 <$ p-value < 0.05; Fail to reject H_0 at $\alpha = 0.01$

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- Step 1 — Hypotheses: H_0: $\mu = 5$ days, H_1: $\mu > 5$ days (right-tailed).
- Step 2 — Conditions: Random sample ✓; $n = 36 \geq 30$, so normality assumed ✓; independent ✓.
- Step 3 — Test statistic: df = 35; $t = (5.8 - 5) / (2.4/\sqrt{36}) = 0.8 / 0.4 = 2.00$.
- Step 4 — p-value: For $t(35) = 2.00$ right-tailed, from t-table: t critical at 0.025 = 2.030, at 0.05 = 1.690. Since $1.690 < 2.00 < 2.030$, the p-value is between 0.025 and 0.05.
- Step 5 — Decision: Since p-value > 0.01 (α), fail to reject H_0.
- Conclusion: At $\alpha = 0.01$, there is not sufficient evidence that the drug results in a mean recovery time greater than 5 days. The company's claim cannot be rejected at this significance level.