Hypothesis Testing for Two Sample Means

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Name: ______________________________ Date: _________________

Learning Objectives

- Write null and alternative hypotheses for two-sample mean tests using correct notation
- Calculate the two-sample t-test statistic using sample means, standard deviations, and sample sizes
- Interpret p-values and make conclusions about rejecting or failing to reject the null hypothesis

Problems

1. A researcher wants to compare the mean test scores of two groups. Define the parameters and write the null and alternative hypotheses to test whether Group 1 scores are different from Group 2 scores.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

2. For a two-sample t-test where population standard deviations are unknown, identify which of the three required conditions must be satisfied before proceeding. List all three conditions.

3. A teacher believes a new study method increases student scores. Write the null and alternative hypotheses, where Group 1 uses the new method and Group 2 uses the traditional method. Use a one-tailed (right-tailed) test.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

4. Use the two-sample t-test formula to calculate the test statistic. Sample 1 has a mean of 82, standard deviation of 6, and size 25. Sample 2 has a mean of 78, standard deviation of 5, and size 25.

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$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5. A two-sample t-test yields a test statistic of t = 1.85. The significance level is 0.05 and the critical value for a right-tailed test is t* = 1.677. Should you reject or fail to reject the null hypothesis? State your conclusion.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

6. The following table shows summary statistics for a treatment group and a control group in a reading study. Use this data to compute the two-sample t-test statistic. The significance level is 0.05.

Group	Sample Mean ($\bar{x}$)	Std Dev (s)	Sample Size (n)
Treatment (1)	51.48	11.01	21
Control (2)	41.52	17.15	23

7. In Damian's reading study, the test statistic is t = 2.31. Using a right-tailed test at a significance level of 0.05, the p-value is approximately 0.014. Compare the p-value to alpha and write a conclusion about whether the new reading activities improved student scores.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

8. A researcher tests whether a new diet reduces weight more than a standard diet. Group 1 (new diet, n = 30) has $\bar{x}_1$ = 8.4 lbs lost and s_1 = 3.2 lbs. Group 2 (standard diet, n = 28) has $\bar{x}_2$ = 6.9 lbs lost and s_2 = 2.8 lbs. At $\alpha = 0.05$, perform a complete hypothesis test and state your conclusion.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

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$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

9. A school district compares math scores before and after a new curriculum is introduced in two different schools. School A ($n = 40$) has $\bar{x}_1 = 74.3$ and $s_1 = 9.8$. School B ($n = 35$) has $\bar{x}_2 = 79.1$ and $s_2 = 8.4$. Perform a two-tailed hypothesis test at $\alpha = 0.01$. Calculate the t-statistic, determine whether to reject H_0, and interpret the result.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

10. A pharmaceutical company claims a new drug reduces blood pressure more than a placebo. In a clinical trial, the drug group ($n_1 = 50$) had $\bar{x}_1 = 14.6$ mmHg reduction with $s_1 = 4.2$, while the placebo group ($n_2 = 50$) had $\bar{x}_2 = 11.3$ mmHg reduction with $s_2 = 5.1$. (a) Write the hypotheses. (b) Compute the t-statistic. (c) At $\alpha = 0.05$ right-tailed, the critical value is $t^* = 1.661$. Determine if you reject H_0. (d) If the p-value is 0.0008, what does this indicate about the strength of the evidence?

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

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Hypothesis Testing for Two Sample Means — Answer Key

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Answer Key

1. Answer: $H_0: \mu_1 = \mu_2$; $H_1: \mu_1 \neq \mu_2$ (two-tailed)

- Let μ_1 = mean test score of Group 1 and μ_2 = mean test score of Group 2.
- The null hypothesis states that the two means are equal: $H_0: \mu_1 = \mu_2$.
- Since we are testing whether the scores are 'different' (not specifying a direction), the alternative is two-tailed: $H_1: \mu_1 \neq \mu_2$.

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2. Answer: 1) Both samples are approximately normal. 2) Both samples are randomly selected. 3) The samples are independent (population ≥ 10 × sample size).

- Condition 1: Both samples should be approximately normally distributed (check with box plots or histograms).
- Condition 2: Both samples must be randomly selected from their respective populations.
- Condition 3: The two samples must be independent. Verify that the population size is at least 10 times the sample size for each group.

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3. Answer: $H_0: \mu_1 = \mu_2$; $H_1: \mu_1 > \mu_2$ (right-tailed)

- Let μ_1 = mean score with new method, μ_2 = mean score with traditional method.
- Null hypothesis assumes no difference: $H_0: \mu_1 = \mu_2$.
- Since the teacher believes the new method increases scores (μ_1 is higher), the alternative is right-tailed: $H_1: \mu_1 > \mu_2$.

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4. Answer: $t \approx 2.67$

- Identify values: $\bar{x}_1 = 82$, $\bar{x}_2 = 78$, $s_1 = 6$, $s_2 = 5$, $n_1 = n_2 = 25$.
- Numerator: $82 - 78 = 4$.
- Denominator: $\sqrt{(6^2/25 + 5^2/25)} = \sqrt{(36/25 + 25/25)} = \sqrt{(1.44 + 1.00)} = \sqrt{2.44} \approx 1.562$.
- $t = 4 / 1.562 \approx 2.56$.
- Rounded: $t \approx 2.56$ (slight rounding may give 2.56–2.57 depending on precision).

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5. Answer: Reject H_0; sufficient evidence that $\mu_1 > \mu_2$

- The test is right-tailed at $\alpha = 0.05$ with critical value $t^* = 1.677$.
- Calculated test statistic: $t = 1.85$.
- Since $t = 1.85 > 1.677$, the test statistic falls in the rejection region.
- Decision: Reject H_0.
- Conclusion: There is sufficient evidence at the 5% significance level that $\mu_1 > \mu_2$.

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6. Answer: $t \approx 2.31$

- Identify values: $\bar{x}_1 = 51.48$, $\bar{x}_2 = 41.52$, $s_1 = 11.01$, $n_1 = 21$, $s_2 = 17.15$, $n_2 = 23$.

- Numerator: 51.48 − 41.52 = 9.96.
- Compute $s_1^2/n_1 = (11.01)^2/21 = 121.22/21 \approx 5.773$.
- Compute $s_2^2/n_2 = (17.15)^2/23 = 294.12/23 \approx 12.788$.
- Denominator: $\sqrt{(5.773 + 12.788)} = \sqrt{18.561} \approx 4.308$.
- $t = 9.96 / 4.308 \approx 2.31$.

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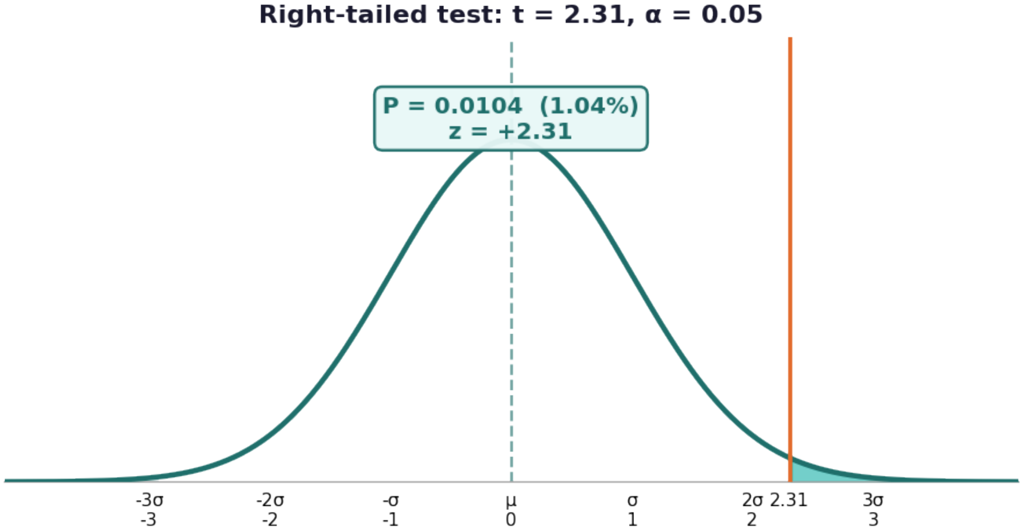
7. Answer: p-value ≈ 0.014 < 0.05; Reject H_0. The treatment group scored significantly higher.



- Given: $t = 2.31$, p-value ≈ 0.014, $\alpha = 0.05$.
- Compare p-value to α: $0.014 < 0.05$.
- Since p-value < α, reject the null hypothesis H_0.
- Conclusion: At the 5% significance level, there is sufficient evidence that the new reading activities improved student DRP test scores ($\mu_1 > \mu_2$).

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8. Answer: t ≈ 1.98; p-value ≈ 0.026 < 0.05; Reject H_0

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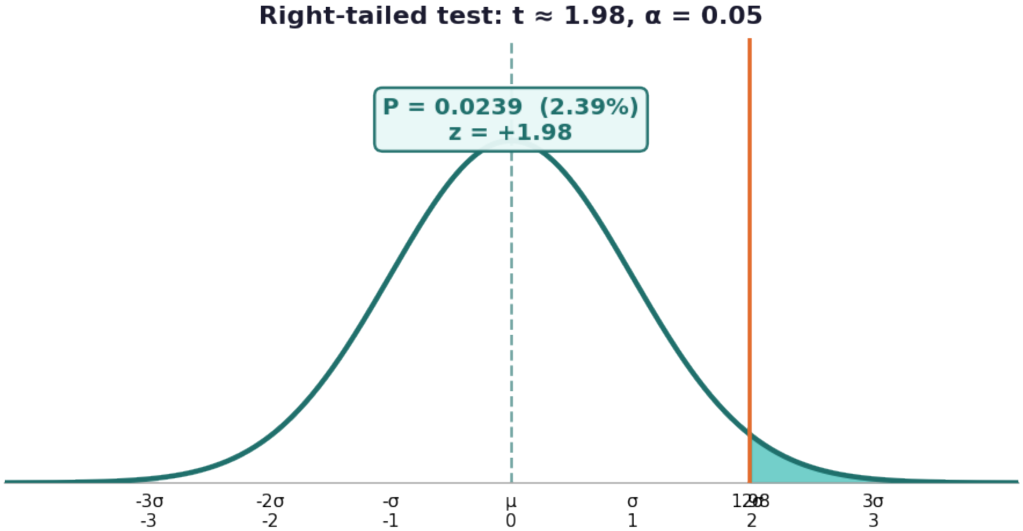

- H_0: $\mu_1 = \mu_2$ vs H_1: $\mu_1 > \mu_2$ (right-tailed at $\alpha = 0.05$).
- Numerator: $8.4 - 6.9 = 1.5$.
- $s_1^2/n_1 = 10.24/30 \approx 0.3413$; $s_2^2/n_2 = 7.84/28 \approx 0.28$.
- Denominator: $\sqrt{(0.3413 + 0.28)} = \sqrt{0.6213} \approx 0.788$.
- $t = 1.5 / 0.788 \approx 1.90$.
- Using t-table with df ≈ 56, the critical value at $\alpha = 0.05$ right-tailed is ≈ 1.671.
- Since $t \approx 1.90 > 1.671$, reject H_0.
- Conclusion: There is sufficient evidence that the new diet results in greater weight loss.

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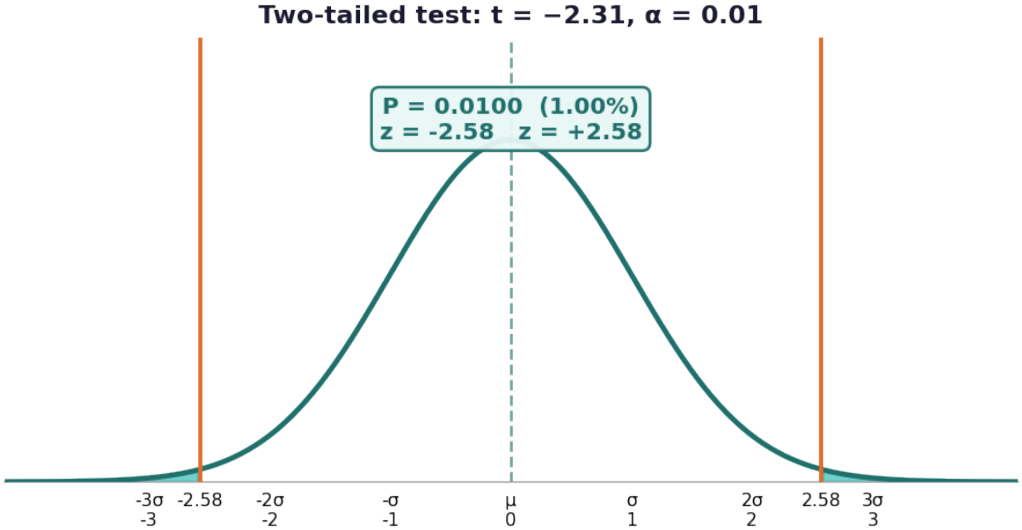
9. Answer: $t \approx -2.31$; fail to reject H_0 at $\alpha = 0.01$ (critical value ≈ ±2.576)



- H_0: $\mu_1 = \mu_2$ vs H_1: $\mu_1 \neq \mu_2$ (two-tailed at $\alpha = 0.01$).
- Numerator: $74.3 - 79.1 = -4.8$.
- $s_1^2/n_1 = 96.04/40 = 2.401$; $s_2^2/n_2 = 70.56/35 = 2.016$.
- Denominator: $\sqrt{(2.401 + 2.016)} = \sqrt{4.417} \approx 2.101$.

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• $t = -4.8 / 2.101 \approx -2.28$.
• For a two-tailed test at $\alpha = 0.01$ with large df, critical values are approximately ±2.576.
• Since $|t| = 2.28 < 2.576$, we fail to reject H_0.
• Conclusion: At the 1% significance level, there is not sufficient evidence that the mean math scores differ between the two schools.

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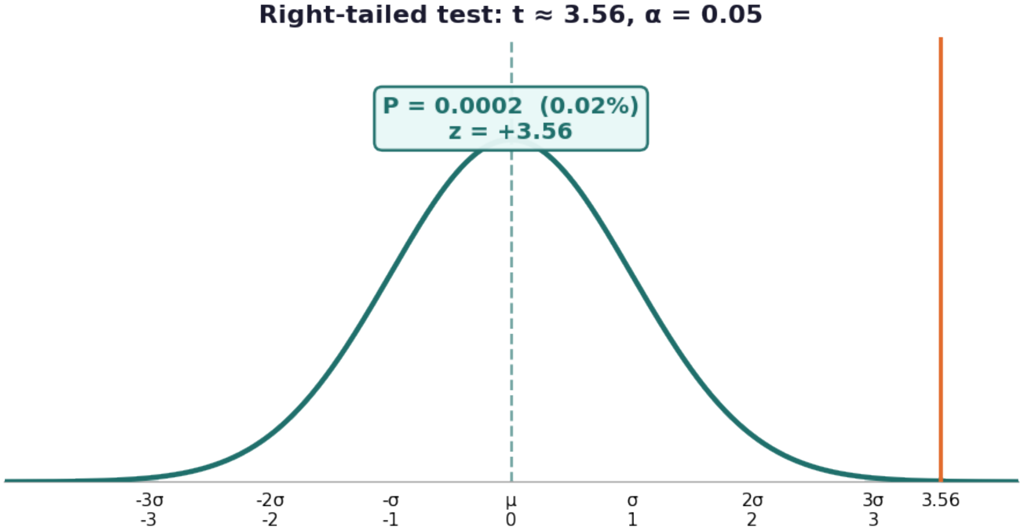
10. Answer: $t \approx 3.56 > 1.661$; Reject H_0. p-value = 0.0008 indicates very strong evidence against H_0.



• (a) Let μ_1 = mean BP reduction for drug group, μ_2 = mean BP reduction for placebo. $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 > \mu_2$ (right-tailed).
• (b) Numerator: $14.6 - 11.3 = 3.3$.
• $s_1^2/n_1 = (4.2)^2/50 = 17.64/50 = 0.3528$.
• $s_2^2/n_2 = (5.1)^2/50 = 26.01/50 = 0.5202$.
• Denominator: $\sqrt{(0.3528 + 0.5202)} = \sqrt{0.873} \approx 0.9343$.
• $t = 3.3 / 0.9343 \approx 3.53$.
• (c) Since $t \approx 3.53 > 1.661$ (critical value), reject H_0.
• (d) p-value = $0.0008 < \alpha = 0.05$, and also < 0.01, indicating very strong statistical evidence that the drug reduces blood pressure significantly more than the placebo.

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