Hypothesis Testing: Null & Alternative Hypotheses

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Name: ______________________________ Date: _________________

Learning Objectives

- Identify and write null and alternative hypotheses for a given claim
- Distinguish between one-tailed (left/right) and two-tailed hypothesis tests
- Calculate and interpret z-test statistics for population means and proportions

Problems

1. A bottling company claims each bottle contains exactly 16 ounces of water. Write the null and alternative hypotheses to test whether the bottles are filled at exactly 16 ounces (two-tailed test). Use μ to represent the mean fill amount.

H_0: $\mu = 16$

H_1: $\mu \neq 16$

2. A tire manufacturer claims its tires last an average of 50,000 miles. A consumer group believes the tires last fewer miles than claimed. Write the null and alternative hypotheses to test the consumer group's claim.

H_0: $\mu = 50000$

H_1: $\mu < 50000$

3. A new fertilizer is advertised to increase the average crop yield above the current average of 120 bushels per acre. A farmer wants to test if the new fertilizer actually increases yield. Write the appropriate null and alternative hypotheses.

H_0: $\mu = 120$

H_1: $\mu > 120$

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4. A school district claims that 65% of its students pass the state math exam. A researcher suspects the actual pass rate is different from 65%. Write the null and alternative hypotheses using p to represent the population proportion.

H_0: $p = 0.65$

H_1: $p \neq 0.65$

5. A pharmaceutical company claims that fewer than 10% of patients experience side effects from a new drug. Identify the null and alternative hypotheses, then state whether this is a left-tailed, right-tailed, or two-tailed test.

H_0: $p = 0.10$

H_1: $p < 0.10$

6. A coffee shop claims its average wait time is 4 minutes. A manager samples 40 customers and finds a sample mean of 4.6 minutes with a known population standard deviation of 1.5 minutes. Write the hypotheses and calculate the z-test statistic. Use α = 0.05 for a two-tailed test.

H_0: $\mu = 4$

H_1: $\mu \neq 4$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

7. A gym owner claims that members exercise an average of at least 5 hours per week. A sample of 36 members shows a mean of 4.5 hours with a known population standard deviation of 1.2 hours. At α = 0.05, test the claim that the mean is less than 5 hours. Write the hypotheses, calculate the test statistic, and state your conclusion.

H_0: $\mu = 5$

H_1: $\mu < 5$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

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8. A university reports that 72% of its graduates find employment within 6 months. A career services analyst samples 200 recent graduates and finds that 156 are employed. At α = 0.05, test whether the true proportion of employed graduates differs from the reported 72%. Write the hypotheses, compute the z-test statistic, and make a conclusion.

$H_0: p = 0.72$

$H_1: p \neq 0.72$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

9. A researcher believes that students who take a prep course score higher than the national average of 510 on a standardized test. A random sample of 50 students who took the prep course had a mean score of 524 with a known population standard deviation of 45. At α = 0.01, conduct a full hypothesis test: state hypotheses, compute the test statistic, identify the critical value, and state a conclusion.

$H_0: \mu = 510$

$H_1: \mu > 510$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

10. A health organization states that 30% of adults in a city are physically inactive. A public health researcher suspects the true proportion is higher. She surveys 350 adults and finds 126 are physically inactive. At α = 0.05, conduct a complete hypothesis test for population proportion: state the hypotheses, calculate the z-test statistic, determine the p-value interpretation, compare to the critical value, and state a conclusion.

$H_0: p = 0.30$

$H_1: p > 0.30$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

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Hypothesis Testing: Null & Alternative Hypotheses — Answer Key

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Answer Key

1. Answer: $H_0: \mu = 16$ $H_1: \mu \neq 16$

$H_0: \mu = 16$

$H_1: \mu \neq 16$

- The null hypothesis always contains an equality (=).
- The claim is $\mu = 16$, so $H_0: \mu = 16$.
- We want to test if it differs in either direction, so the alternative is two-tailed.
- $H_1: \mu \neq 16$.

2. Answer: $H_0: \mu = 50000$ $H_1: \mu < 50000$ (left-tailed)

$H_0: \mu = 50000$

$H_1: \mu < 50000$

- The null hypothesis states the manufacturer's claim is true: $H_0: \mu = 50000$.
- The consumer group believes tires last fewer miles, meaning μ is less than the claimed value.
- This is a left-tailed test: $H_1: \mu < 50000$.

3. Answer: $H_0: \mu = 120$ $H_1: \mu > 120$ (right-tailed)

$H_0: \mu = 120$

$H_1: \mu > 120$

- The null hypothesis assumes no change from the current average: $H_0: \mu = 120$.
- The advertised claim is that yield increases, so we test if μ is greater than 120.
- This is a right-tailed test: $H_1: \mu > 120$.

4. Answer: $H_0: p = 0.65$ $H_1: p \neq 0.65$ (two-tailed)

$H_0: p = 0.65$

$H_1: p \neq 0.65$

- The null hypothesis states the district's claim is true: $H_0: p = 0.65$.
- The researcher suspects the rate is different (not just higher or lower), so it is two-tailed.
- $H_1: p \neq 0.65$.

5. Answer: $H_0: p = 0.10$ $H_1: p < 0.10$ — Left-tailed test

H_0: p = 0.10

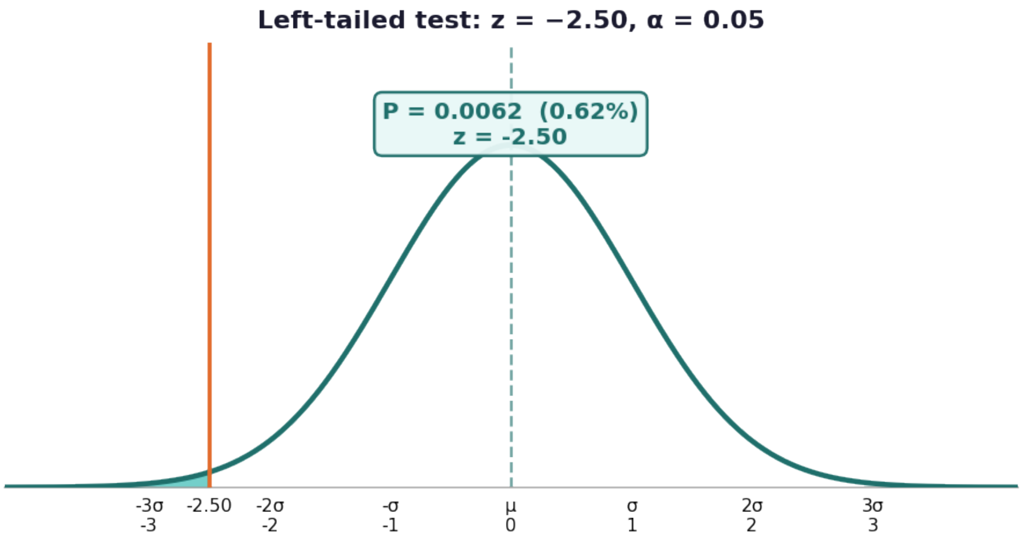
H_1: p < 0.10

- The null hypothesis sets the proportion equal to the claimed value: H_0: p = 0.10.
- The company claims fewer than 10%, meaning p is less than 0.10.
- H_1: p < 0.10 — this is a left-tailed test because the alternative uses '<'.

6. Answer: z ≈ 2.53; Reject H_0

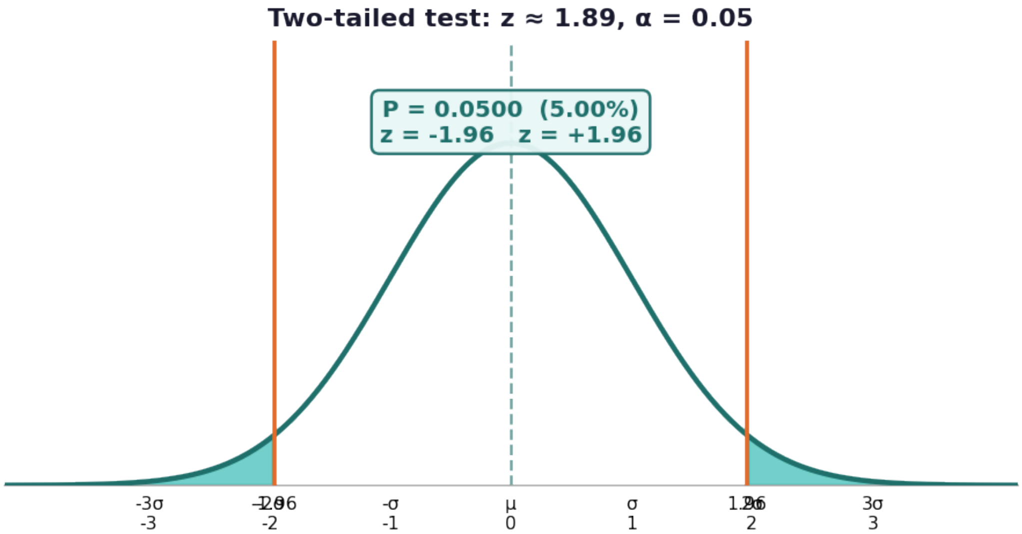
- H_0: $\mu = 4$, H_1: $\mu \neq 4$ (two-tailed)
- Given: $\bar{x} = 4.6$, $\mu_0 = 4$, $\sigma = 1.5$, $n = 40$
- $z = (4.6 - 4) / (1.5 / \sqrt{40}) = 0.6 / (1.5 / 6.325) = 0.6 / 0.2372 \approx 2.53$
- Critical values for two-tailed $\alpha = 0.05$: $z = \pm 1.96$
- Since $|2.53| > 1.96$, we reject H_0.
- Conclusion: There is sufficient evidence that the mean wait time differs from 4 minutes.

7. Answer: z = −2.50; Reject H_0



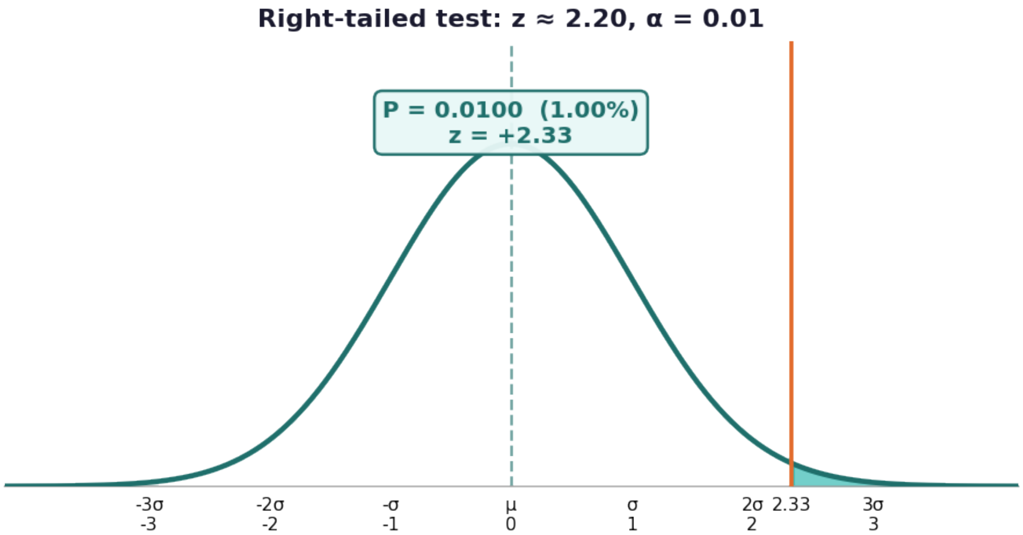
- H_0: $\mu = 5$, H_1: $\mu < 5$ (left-tailed)
- Given: $\bar{x} = 4.5$, $\mu_0 = 5$, $\sigma = 1.2$, $n = 36$
- $z = (4.5 - 5) / (1.2 / \sqrt{36}) = -0.5 / (1.2 / 6) = -0.5 / 0.2 = -2.50$
- Critical value for left-tailed $\alpha = 0.05$: $z = -1.645$
- Since $-2.50 < -1.645$, we reject H_0.
- Conclusion: There is sufficient evidence that the mean exercise time is less than 5 hours.

8. Answer: z ≈ 1.49; Fail to reject H_0



- H_0: p = 0.72, H_1: p ≠ 0.72 (two-tailed)
- Sample proportion: $\hat{p}$ = 156/200 = 0.78
- Standard error = $\sqrt{0.72 \times 0.28 / 200} = \sqrt{0.2016/200} = \sqrt{0.001008} \approx 0.03175$
- $z = (0.78 - 0.72) / 0.03175 \approx 0.06 / 0.03175 \approx 1.89$
- Critical values for two-tailed $\alpha = 0.05$: $z = \pm 1.96$
- Since $|1.89| < 1.96$, we fail to reject H_0.
- Conclusion: There is insufficient evidence that the proportion differs from 72%.

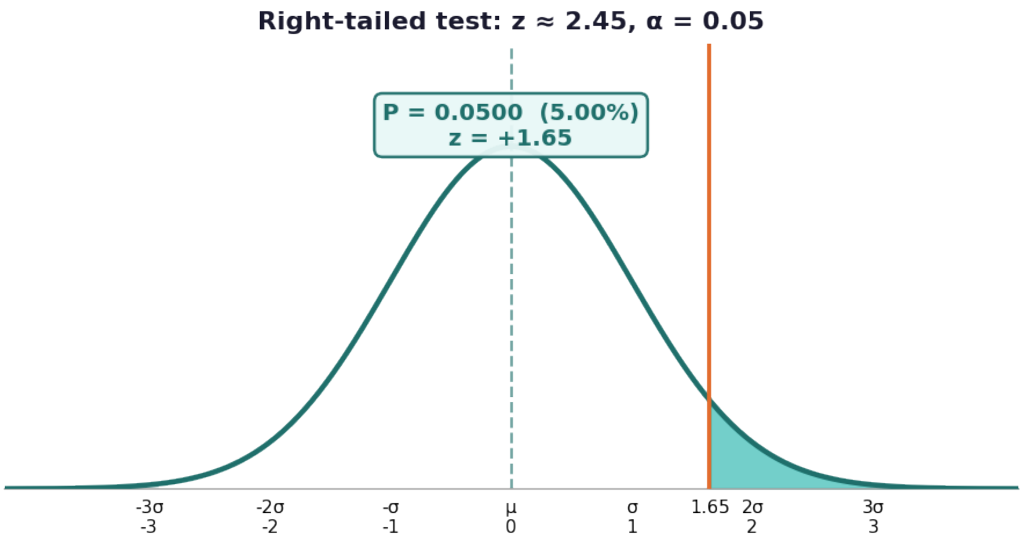
9. Answer: z ≈ 2.20; Reject H_0 at α = 0.01



- H_0: μ = 510, H_1: μ > 510 (right-tailed)
- Given: $\bar{x}$ = 524, μ_0 = 510, σ = 45, n = 50
- $z = (524 - 510) / (45 / \sqrt{50}) = 14 / (45 / 7.071) = 14 / 6.364 \approx 2.20$
- Critical value for right-tailed $\alpha = 0.01$: $z = 2.326$
- Since $2.20 < 2.326$, we fail to reject H_0 at $\alpha = 0.01$.

• Conclusion: At the 1% significance level, there is insufficient evidence that the prep course raises scores above 510.

10. Answer: $z \approx 1.83$; Fail to reject H_0



- H_0: $p = 0.30$, H_1: $p > 0.30$ (right-tailed)
- Sample proportion: $\hat{p} = 126 / 350 = 0.36$
- Standard error = $\sqrt{(0.30 \times 0.70 / 350)} = \sqrt{(0.21 / 350)} = \sqrt{0.0006} \approx 0.02449$
- $z = (0.36 - 0.30) / 0.02449 = 0.06 / 0.02449 \approx 2.45$
- Critical value for right-tailed $\alpha = 0.05$: $z = 1.645$
- Since $2.45 > 1.645$, we reject H_0.
- Conclusion: At the 5% significance level, there is sufficient evidence that more than 30% of adults in the city are physically inactive.

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