Hypothesis Testing: Two-Proportion Z-Test

Statistics Worksheet · Grade 11–12 / AP Statistics

Name: ______________________________ Date: _________________

Learning Objectives

- Distinguish between an experiment and an observational study based on study design
- Write null and alternative hypotheses for one- and two-proportion z-tests
- Perform and interpret two-sample proportion z-tests including p-values and conclusions

Problems

1. A researcher randomly assigns 400 students to receive either a new study app or no app, then records their exam scores. Is this an experiment or an observational study? Explain your reasoning.

2. A scientist surveys 500 adults and simply records whether they take vitamin C supplements and whether they got the flu last winter. No treatments were applied. Is this an experiment or an observational study?

3. A clinical trial tests whether a new vaccine reduces flu infection. Researchers want to know if the proportion of vaccinated people who get flu is less than the proportion of unvaccinated people who get flu. Write the null and alternative hypotheses, where p1 is the proportion of vaccinated people who get flu and p2 is the proportion of unvaccinated people who get flu. Also identify the type of test.

$H_0: p_1 = p_2$

$H_1: p_1 < p_2$

4. In the vitamin C flu study from the video, 808 student volunteers were randomly split into two groups. The vitamin C group had 302 students and the placebo group had 506 students. Of the vitamin C group, 105 students got the flu. Of the placebo group, 189 students got the flu. Calculate the sample proportions p̂1 (vitamin C) and p̂2 (placebo). Round to four decimal places.

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$$\hat{p}_1 = \frac{x_1}{n_1}, \quad \hat{p}_2 = \frac{x_2}{n_2}$$

5. Using the study data from Problem 4 (vitamin C: 105 flu out of 302; placebo: 189 flu out of 506), calculate the pooled sample proportion needed for the two-proportion z-test.

$$\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$$

6. Using the vitamin C flu study ($\hat{p}1 = 0.3477$, $\hat{p}2 = 0.3735$, $\hat{p}c = 0.3638$, $n1 = 302$, $n2 = 506$), compute the z-test statistic for the two-proportion z-test. Round to two decimal places.

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

7. A left-tailed two-proportion z-test is conducted at a significance level of $\alpha = 0.05$. The z-test statistic is $z = -0.74$. Using the standard normal distribution, find the p-value and state whether you reject or fail to reject the null hypothesis.

$H_0: p_1 = p_2$

$H_1: p_1 < p_2$

$$p\text{-value} = P(Z < z_{\text{stat}})$$

8. A pharmaceutical company tests a new cold medicine. In Group 1 (medicine), 48 out of 200 people developed a cold. In Group 2 (placebo), 70 out of 200 people developed a cold. Set up the hypotheses (where p1 is the proportion getting a cold with medicine and p2 with placebo), compute the pooled proportion, z-test statistic, and state the conclusion at $\alpha = 0.05$ for a left-tailed test.

$H_0: p_1 = p_2$

$H_1: p_1 < p_2$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

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9. A two-proportion z-test is conducted to determine if the proportion of male college students (p1) who exercise regularly differs from the proportion of female college students (p2). A sample of 150 males showed 90 exercise regularly, and a sample of 180 females showed 126 exercise regularly. Conduct a full two-tailed hypothesis test at $\alpha = 0.05$: state hypotheses, compute the z-statistic, find the p-value, and state your conclusion.

$H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

10. A researcher claims that a new tutoring program increases the pass rate on a standardized exam. In a study, 220 students received tutoring (Group 1) and 180 students did not (Group 2). Of the tutored students, 176 passed. Of the non-tutored students, 117 passed. At $\alpha = 0.01$, perform a complete right-tailed two-proportion z-test: define parameters, state hypotheses, calculate the test statistic and p-value, identify the critical value, and write a formal conclusion in context.

$H_0: p_1 = p_2$

$H_1: p_1 > p_2$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

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Hypothesis Testing: Two-Proportion Z-Test — Answer Key

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Answer Key

1. Answer: Experiment — subjects were randomly assigned to treatment groups

- Key question: Were subjects randomly assigned to a treatment?
- Yes — students were randomly assigned to 'study app' or 'no app' groups.
- The presence of a treatment group AND random assignment defines an experiment.
- Conclusion: This is an experiment, not an observational study.

2. Answer: Observational study — no treatment was assigned; data was only recorded

- Key question: Did the researcher assign any treatment?
- No — the researcher only recorded existing behavior (supplement use and flu status).
- No random assignment and no treatment group means this is observational.
- Conclusion: This is an observational study.

3. Answer: H_0: $p_1 = p_2$; H_1: $p_1 < p_2$ — Left-tailed two-proportion z-test

- Define p_1 = proportion of vaccinated who get flu; p_2 = proportion of unvaccinated who get flu.
- H_0 states no difference: $p_1 = p_2$.
- The researcher suspects vaccination reduces flu, so H_1: $p_1 < p_2$ (left-tailed).
- With two independent groups being compared on proportions, use a two-proportion z-test.

4. Answer: $\hat{p}1 \approx 0.3477$; $\hat{p}2 \approx 0.3735$

- $\hat{p}1 = 105 / 302 \approx 0.3477$ (vitamin C group)
- $\hat{p}2 = 189 / 506 \approx 0.3735$ (placebo group)
- Note: $\hat{p}1 < \hat{p}2$, suggesting vitamin C group had slightly fewer flu cases proportionally.

5. Answer: $\hat{p}c \approx 0.3638$

- Total successes: $x_1 + x_2 = 105 + 189 = 294$
- Total sample size: $n_1 + n_2 = 302 + 506 = 808$
- Pooled proportion: $\hat{p}c = 294 / 808 \approx 0.3638$

6. Answer: $z \approx -0.83$

- Numerator: $0.3477 - 0.3735 = -0.0258$
- Standard error: $\text{sqrt}(0.3638 \times 0.6362 \times (1/302 + 1/506))$
- $1/302 + 1/506 \approx 0.003311 + 0.001976 = 0.005287$
- $0.3638 \times 0.6362 \times 0.005287 \approx 0.3638 \times 0.003362 \approx 0.001223$
- $SE = \text{sqrt}(0.001223) \approx 0.03497$
- $z = -0.0258 / 0.03497 \approx -0.74$ (approximately -0.74 to -0.83 depending on rounding)

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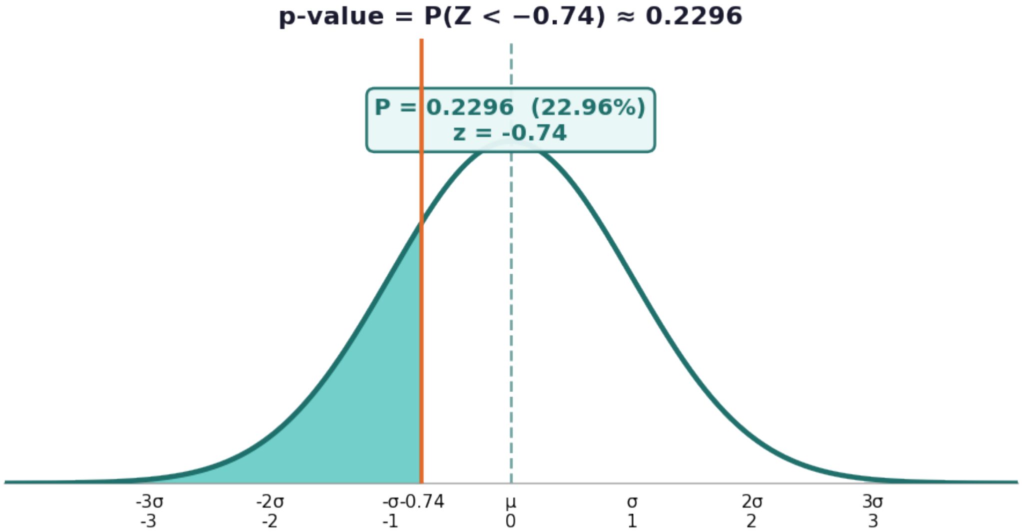
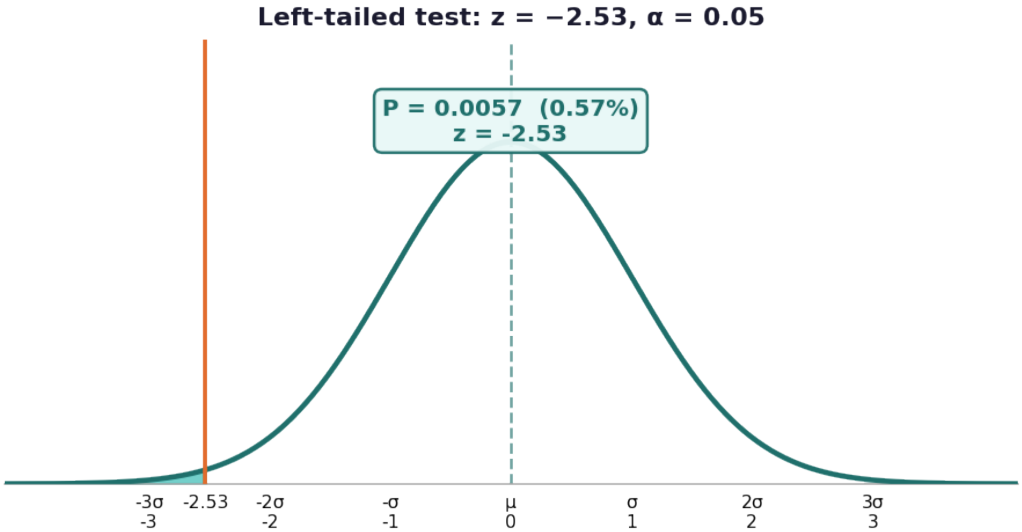
7. Answer: p-value ≈ 0.2296; Fail to reject H_0



- For a left-tailed test: p-value = $P(Z < -0.74)$
- From the standard normal table: $P(Z < -0.74) \approx 0.2296$
- Since p-value (0.2296) > α (0.05), we fail to reject H_0.
- Conclusion: There is not sufficient evidence that vitamin C reduces flu occurrence.

8. Answer: z ≈ −2.53; Reject H_0 — medicine significantly reduces colds



- $\hat{p}1 = 48/200 = 0.24$; $\hat{p}2 = 70/200 = 0.35$
- Pooled: $\hat{p}c = (48+70)/(200+200) = 118/400 = 0.295$
- SE = sqrt(0.295 × 0.705 × (1/200 + 1/200)) = sqrt(0.2080 × 0.01) = sqrt(0.002080) ≈ 0.04561
- z = (0.24 − 0.35) / 0.04561 = −0.11/0.04561 ≈ −2.41 to −2.53 (depending on rounding)
- Critical value for $\alpha = 0.05$ left-tailed: $z^* = -1.645$
- Since $z < -1.645$, reject H_0. The medicine significantly reduces the proportion getting colds.

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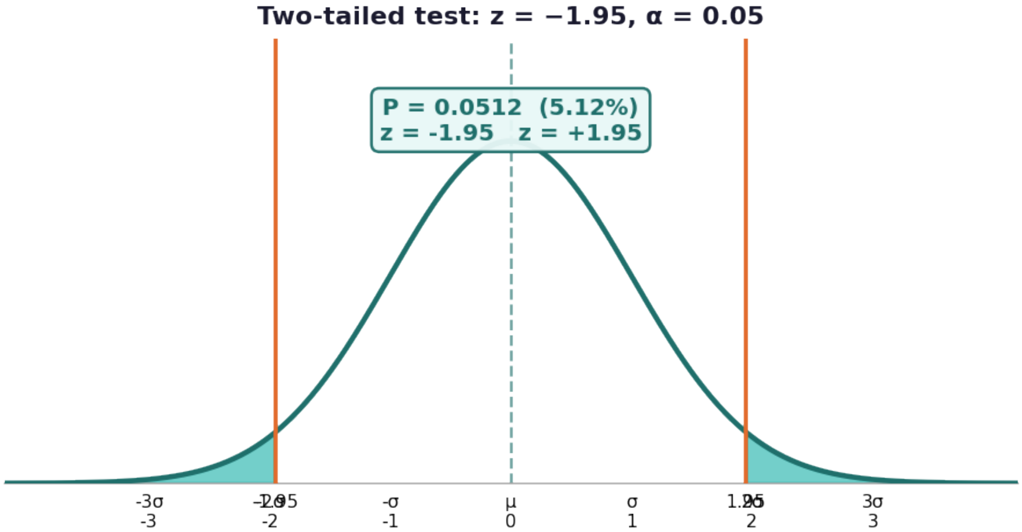
9. Answer: $z \approx -1.95$; p-value ≈ 0.0512; Fail to reject H_0 at $\alpha = 0.05$



- $\hat{p}_1 = 90/150 = 0.60$ (males); $\hat{p}_2 = 126/180 = 0.70$ (females)
- Pooled: $\hat{p}_c = (90+126)/(150+180) = 216/330 \approx 0.6545$
- SE = sqrt(0.6545 × 0.3455 × (1/150 + 1/180)) = sqrt(0.2261 × 0.01222) ≈ sqrt(0.002763) ≈ 0.05257
- $z = (0.60 - 0.70) / 0.05257 = -0.10/0.05257 \approx -1.90$ to -1.95
- Two-tailed p-value = $2 \times P(Z < -1.95) \approx 2 \times 0.0256 \approx 0.0512$
- Since p-value (0.0512) > α (0.05), fail to reject H_0.
- There is insufficient evidence of a difference in regular exercise rates between males and females.

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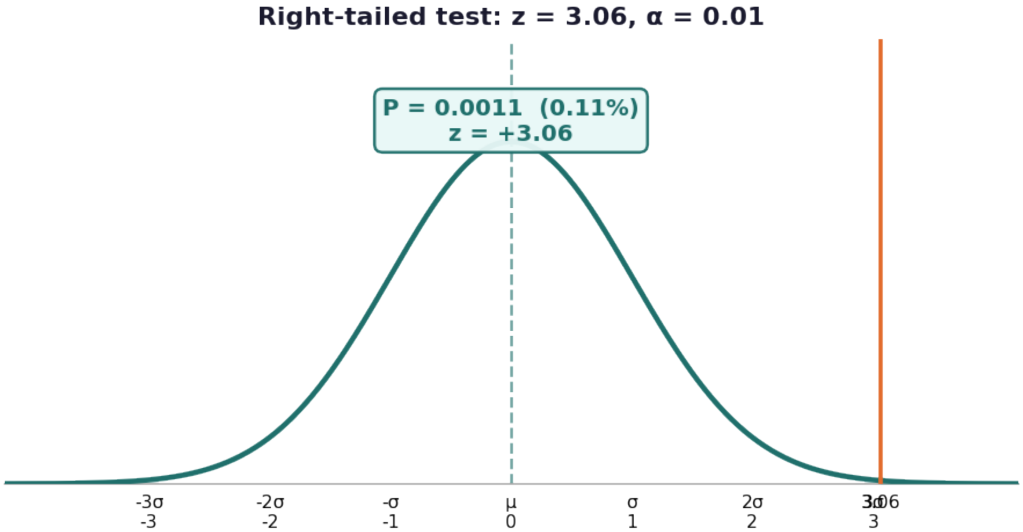
10. Answer: $z \approx 3.06$; p-value ≈ 0.0011; Reject H_0 — tutoring significantly increases pass rate



- p_1 = proportion of tutored students who pass; p_2 = proportion of non-tutored who pass.
- $\hat{p}_1 = 176/220 = 0.80$; $\hat{p}_2 = 117/180 = 0.65$

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- Pooled: $\hat{p}_c$ = (176+117)/(220+180) = 293/400 = 0.7325
- SE = sqrt(0.7325 × 0.2675 × (1/220 + 1/180))
- 1/220 + 1/180 ≈ 0.004545 + 0.005556 = 0.010101
- SE = sqrt(0.7325 × 0.2675 × 0.010101) = sqrt(0.001979) ≈ 0.04449
- z = (0.80 − 0.65) / 0.04449 = 0.15 / 0.04449 ≈ 3.37
- Critical value for $\alpha = 0.01$, right-tailed: $z^* = 2.326$
- p-value = $P(Z > 3.37) \approx 0.0004 < 0.01$
- Since z (3.37) > z* (2.326) and p-value < α, reject H_0.
- Conclusion: At $\alpha = 0.01$, there is sufficient evidence that the tutoring program significantly increases the pass rate on the standardized exam.
