Matched Pairs Design & Paired t-Test

Statistics Worksheet · AP Statistics / Grade 11–12

Name: ______________________________ Date: _________________

Learning Objectives

- Identify when a matched pairs (paired t-test) design is appropriate and state the null and alternative hypotheses
- Calculate the paired differences, sample mean of differences, and standard deviation of differences, then compute the t-test statistic
- Interpret p-values and t-statistics in context to draw conclusions about paired data at a given significance level

Problems

1. Chelsea is investigating whether a floral-scented mask improves maze-solving speed. She measures each student's time with an unscented mask and then with a scented mask. Explain why this is a matched pairs (paired t-test) design rather than a two-sample independent design.

2. For the matched pairs design, define the parameter of interest and write the null and alternative hypotheses. Chelsea wants to test whether the scented mask has ANY effect (positive or negative) on maze time.

H_0: μ_d = 0

H_1: μ_d ≠ 0

3. The table below shows data for five of Chelsea's 21 students. Calculate the difference (unscented minus scented) for each student and fill in the missing values in the Difference column.

Student	Unscented (sec)	Scented (sec)	Difference (sec)
1	30.60	37.97	
2	48.20	41.50	
3	25.00	22.80	
4	60.10	55.30	

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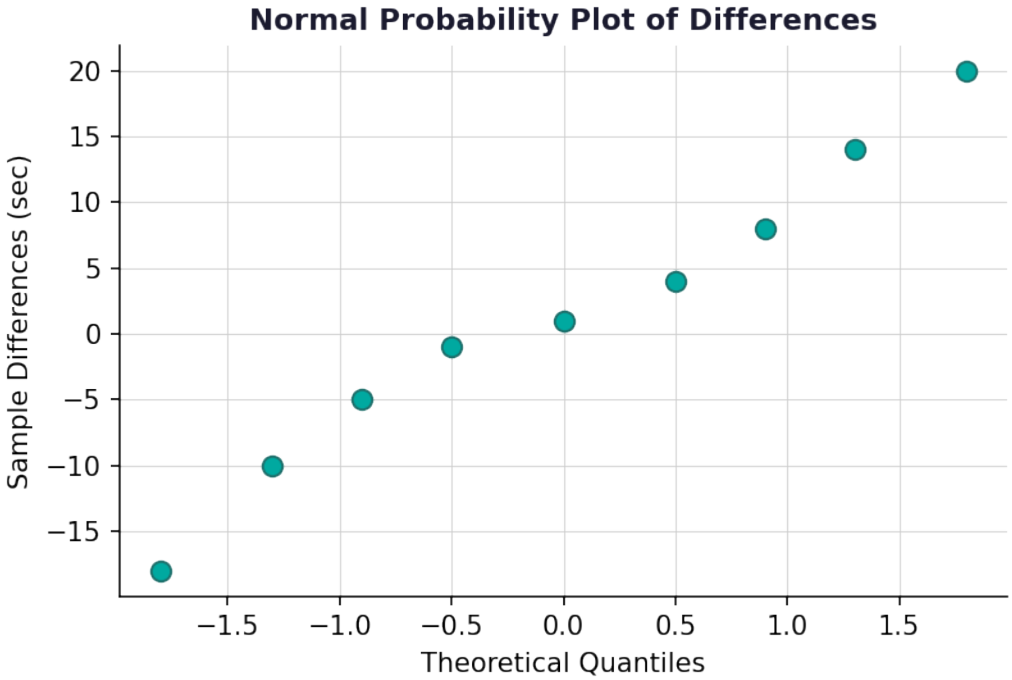
Student	Unscented (sec)	Scented (sec)	Difference (sec)
5	33.45	36.90	

4. In Chelsea's experiment, student 13 had an unscented mask time of 54.47 seconds and a scented mask time of 38.30 seconds. What is the paired difference for student 13, and what does a positive difference tell us about the effect of the scented mask for this student?

$$d_{13} = x_{\text{unscented}} - x_{\text{scented}}$$

5. After collecting data from all 21 students, the normal probability plot of the differences is examined. The points fall close to a straight diagonal line. What condition does this satisfy, and why is it required before conducting a paired t-test?



6. Chelsea found that the mean of the 21 paired differences is 0.9567 seconds with a standard deviation of 12.5479 seconds. Calculate the t-test statistic for the paired t-test. Use 4 decimal places in your answer.

$H_0: \mu_d = 0$

$H_1: \mu_d \neq 0$

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$$t = \frac{\bar{d} - \mu_0}{s_d/\sqrt{n}}$$

7. Using the t-statistic from Problem 6 and 20 degrees of freedom, the two-tailed p-value is approximately 0.730. At a significance level of alpha equals 0.05, state the conclusion of Chelsea's hypothesis test in context.

H_0: μ_d = 0

H_1: μ_d ≠ 0

$$p\text{-value} \approx 0.730 > \alpha = 0.05$$

8. A new researcher repeats Chelsea's study but now believes the scented mask will specifically reduce (improve) maze completion time. The 21 students yield the same statistics: mean difference equals 2.85 seconds and standard deviation of differences equals 10.20 seconds. At alpha equals 0.05, perform a one-tailed paired t-test and state your conclusion.

H_0: μ_d = 0

H_1: μ_d > 0

$$t = \frac{\bar{d} - 0}{s_d/\sqrt{n}}$$

9. A different experiment tests whether background music lowers student anxiety scores. Each of 16 students is given an anxiety test before and after listening to music for 20 minutes. The differences (before minus after) have a mean of 4.8 points and a standard deviation of 7.2 points. The normal probability plot shows the differences are approximately normal. Conduct a full paired t-test at the 5 percent significance level to determine whether music significantly reduces anxiety.

H_0: μ_d = 0

H_1: μ_d > 0

$$t = \frac{\bar{d} - \mu_0}{s_d/\sqrt{n}}$$

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10. In a rigorous follow-up study on scented masks with 25 students, a paired t-test at alpha equals 0.01 yields the following results: mean difference equals 3.12 seconds, standard deviation of differences equals 8.45 seconds. (a) Compute the t-statistic. (b) Find the degrees of freedom. (c) The p-value for a two-tailed test is 0.083. What decision do you make? (d) Would the conclusion change at alpha equals 0.10? Justify your answer.

H_0: μ_d = 0

H_1: μ_d ≠ 0

$$t = \frac{\bar{d} - 0}{s_d/\sqrt{n}}$$

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Matched Pairs Design & Paired t-Test — Answer Key

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Answer Key

1. Answer: Each student serves as their own control; the two measurements are paired within the same individual, removing individual variability.

- In a matched pairs design, each subject is measured under both conditions (unscented then scented).
- The two observations are linked (paired) within the same individual, so they are NOT independent.
- By using differences within each pair, we eliminate between-subject variability, increasing the power of the test.
- This is different from a two-sample test, where two separate, unrelated groups are compared.

2. Answer: μ_d = mean difference (unscented − scented) in maze time; H_0: $\mu_d = 0$; H_1: $\mu_d \neq 0$ (two-tailed)

- Parameter: μ_d = the true mean difference in maze completion time (unscented − scented) for all BHS students.
- H_0: $\mu_d = 0$ → No effect; the scented mask does not change average maze time.
- H_1: $\mu_d \neq 0$ → The scented mask does change average maze time (two-tailed, since 'any effect' is tested).
- A two-tailed test is used because we are looking for improvement OR worsening.

3. Answer: −7.37, 6.70, 2.20, 4.80, −3.45

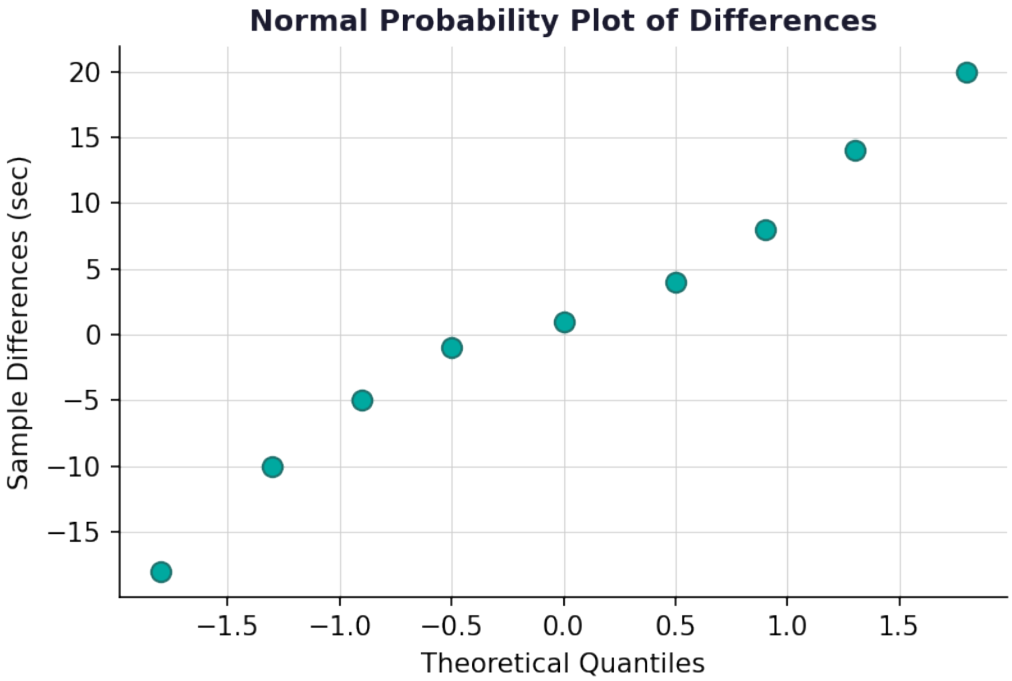
Student	Unscented (sec)	Scented (sec)	Difference (sec)
1	30.60	37.97	−7.37
2	48.20	41.50	6.70
3	25.00	22.80	2.20
4	60.10	55.30	4.80
5	33.45	36.90	−3.45

- Difference = Unscented − Scented for each student.
- Student 1: 30.60 − 37.97 = −7.37 (negative means scented took longer)
- Student 2: 48.20 − 41.50 = 6.70 (positive means scented was faster)
- Student 3: 25.00 − 22.80 = 2.20
- Student 4: 60.10 − 55.30 = 4.80
- Student 5: 33.45 − 36.90 = −3.45

4. Answer: d_{13} = 16.17 sec; a positive difference means the student solved the maze faster while wearing the scented mask.

- $d_{13} = 54.47 - 38.30 = 16.17$ seconds
- A positive difference (unscented − scented > 0) means the unscented time was longer.
- Therefore, the student completed the maze more quickly while wearing the scented mask.
- This suggests the scented mask may have improved performance for student 13.

5. Answer: The points near the diagonal confirm approximate normality of the differences, satisfying the normality condition required for the paired t-test.



- A normal probability plot graphs sample quantiles vs. theoretical normal quantiles.
- If the points fall approximately along a straight line, the data is approximately normally distributed.
- The paired t-test requires that either (a) the differences are normally distributed, or (b) the sample size is large enough ($n \geq 30$) by the Central Limit Theorem.
- With $n = 21$ (< 30), we rely on the normal probability plot to verify normality.
- Since the plot shows points close to the diagonal, the normality condition is satisfied.

6. Answer: $t \approx 0.3496$

- Given: $\bar{x}_d = 0.9567$, $s_d = 12.5479$, $n = 21$, $\mu_0 = 0$
- Standard error: $SE = s_d / \sqrt{n} = 12.5479 / \sqrt{21} = 12.5479 / 4.5826 \approx 2.7381$
- $t = (0.9567 - 0) / 2.7381 \approx 0.3496$
- Degrees of freedom: $df = n - 1 = 21 - 1 = 20$

7. Answer: Fail to reject H_0. There is insufficient evidence at $\alpha = 0.05$ that the scented mask significantly changes maze completion time.

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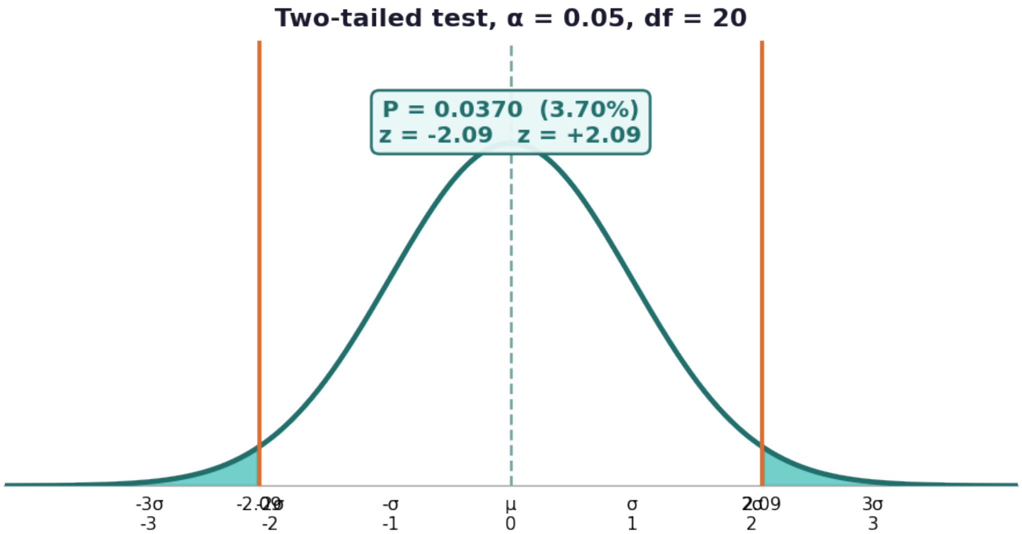

- Decision rule: Reject H_0 if p-value < $\alpha = 0.05$.
- p-value ≈ 0.730 > 0.05, so we fail to reject H_0.
- Alternatively, critical values at $\alpha = 0.05$ (two-tailed, df = 20) are $t^* = \pm 2.086$.
- $t \approx 0.3496$ falls between −2.086 and 2.086, so we fail to reject H_0.
- Conclusion: There is not sufficient evidence to conclude that the floral-scented mask has any effect on students' average maze completion time.

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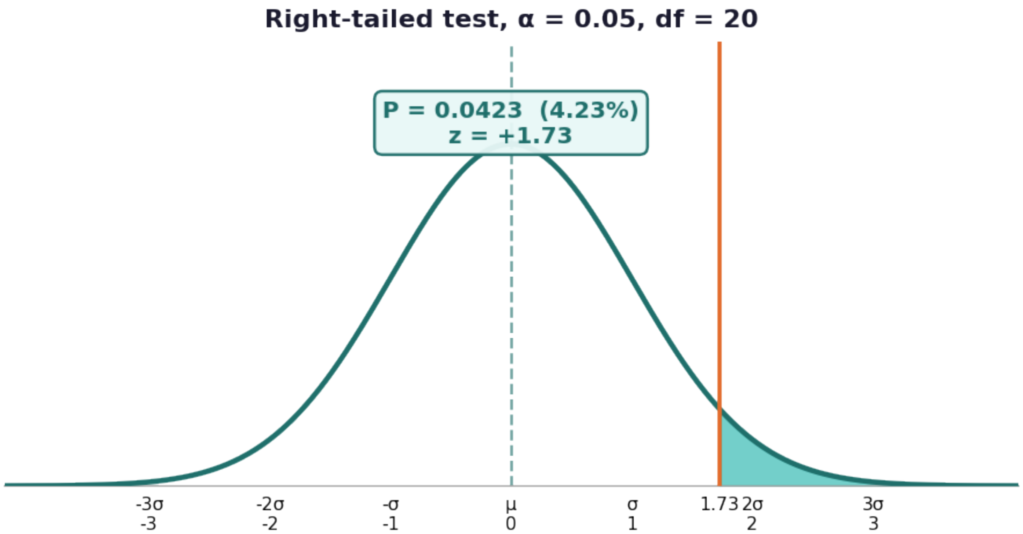
8. Answer: t ≈ 1.282; critical value t* = 1.725; fail to reject H_0. Insufficient evidence the scented mask reduces time.



- H_0: $\mu_d = 0$, H_1: $\mu_d > 0$ (right-tailed, expecting positive differences = faster with scented mask)
- $SE = 10.20 / \sqrt{21} = 10.20 / 4.5826 \approx 2.2257$
- $t = 2.85 / 2.2257 \approx 1.282$
- Critical value: $t^*(\alpha = 0.05, df = 20, \text{right-tailed}) = 1.725$
- Since $t = 1.282 < 1.725$, we fail to reject H_0.

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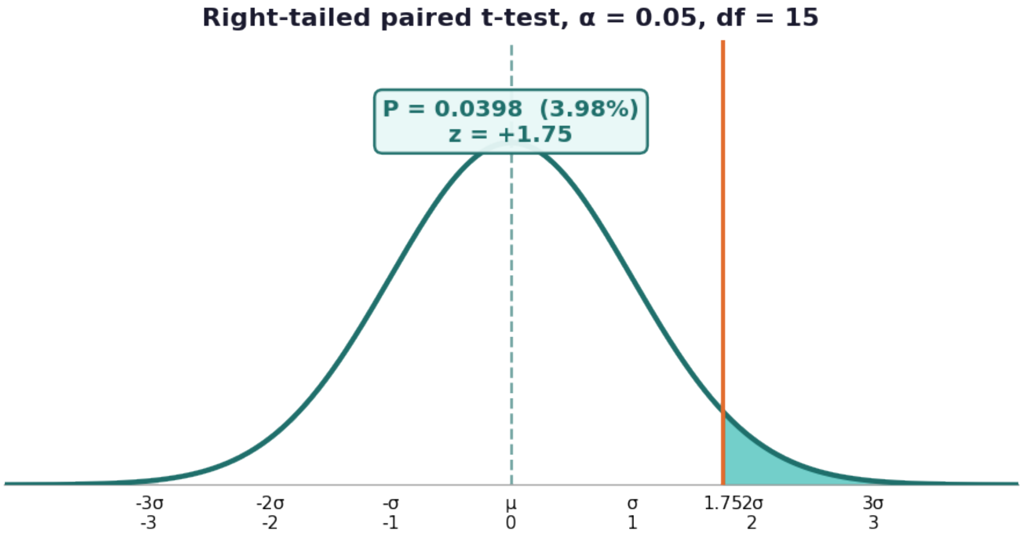
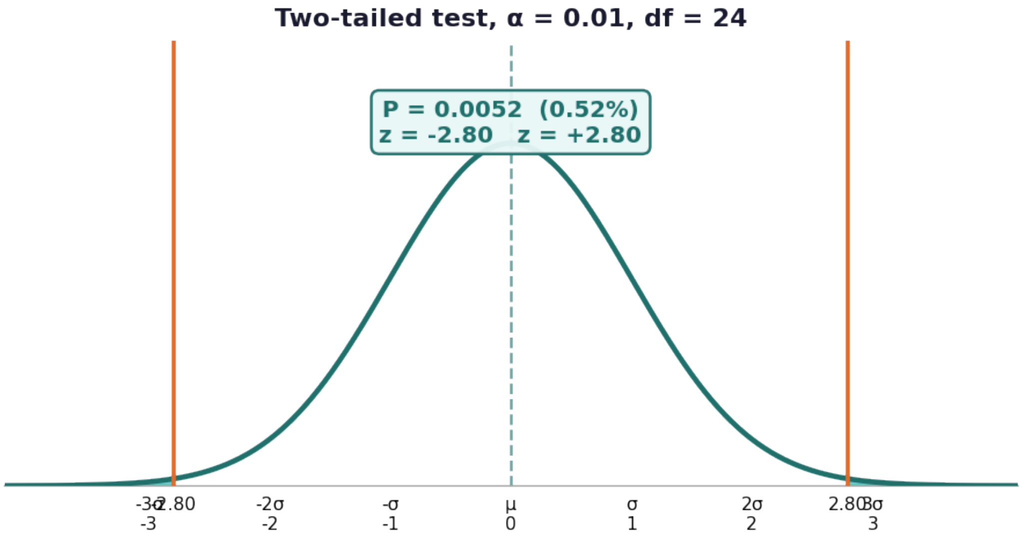
• Conclusion: There is insufficient evidence at α = 0.05 that the scented mask reduces maze completion time.

9. Answer: t ≈ 2.667; critical value t* = 1.753 (df = 15); reject H_0. Music significantly reduces anxiety.



• Parameter: μ_d = true mean reduction in anxiety score (before − after).
• H_0: μ_d = 0, H_1: μ_d > 0 (right-tailed; testing for a reduction).
• Conditions: Random sample, n = 16 ≥ ... normal probability plot confirms normality — conditions met.
• SE = s_d / √n = 7.2 / √16 = 7.2 / 4 = 1.8
• t = (4.8 − 0) / 1.8 = 2.667
• df = 16 − 1 = 15; critical value t*(0.05, 15) = 1.753
• Since t = 2.667 > 1.753, reject H_0.
• Conclusion: There is sufficient evidence at α = 0.05 that listening to music significantly reduces student anxiety scores.

10. Answer: (a) t ≈ 1.845; (b) df = 24; (c) Fail to reject H_0 at α = 0.01; (d) Yes — reject H_0 at α = 0.10 since 0.083 < 0.10.



- (a) SE = 8.45 / √25 = 8.45 / 5 = 1.69; t = 3.12 / 1.69 ≈ 1.845
- (b) df = n − 1 = 25 − 1 = 24
- (c) At $\alpha = 0.01$ (two-tailed): critical values $t^* = \pm 2.797$. Since $|1.845| < 2.797$ AND p-value = 0.083 > 0.01, we fail to reject H_0.
- Conclusion at $\alpha = 0.01$: There is insufficient evidence that the scented mask changes maze time.
- (d) At $\alpha = 0.10$: p-value = 0.083 < 0.10, so we WOULD reject H_0.
- Yes, the conclusion changes — at a less stringent significance level the scented mask effect becomes statistically significant.
- This illustrates how the choice of α affects the outcome of a hypothesis test.

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