Mean and Standard Deviation of a Discrete Random Variable

Statistics Worksheet · Grade 11–12

Name: ________________________________ Date: __________________

Learning Objectives

- Calculate the mean (expected value) of a discrete random variable using the formula $E(X) = \Sigma[x \cdot P(x)]$
- Compute the variance and standard deviation of a discrete random variable using $\sigma^2 = \Sigma[(x - \mu)^2 \cdot P(x)]$
- Interpret the mean and standard deviation in the context of real-world probability distributions

Problems

1. The table below shows the probability distribution of X, the number of heads when a coin is flipped twice. Find the mean (expected value) of X.

x	P(x)	x · P(x)
0	0.25	
1	0.50	
2	0.25	

2. A discrete random variable X has the probability distribution shown below. Verify that the probabilities sum to 1, then find the mean of X.

x	P(x)	x · P(x)
1	0.10	
2	0.20	
3	0.40	
4	0.20	
5	0.10	

3. The number of absences per week X for students has the distribution below. Find the mean number of absences per week.

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x	P(x)	x · P(x)
0	0.60	
1	0.25	
2	0.10	
3	0.05	

4. Using the probability distribution of X below, complete the variance table and find the variance of X. The mean has already been calculated as $\mu = 2$.

x	P(x)	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 \cdot P(x)$
0	0.20			
2	0.50			
4	0.30			

5. Using the variance found in Problem 4 ($\sigma^2 = 2.00$), find the standard deviation of X. Round your answer to four decimal places.

$$\sigma = \sqrt{\sigma^2} = \sqrt{2.00}$$

6. A raffle sells 500 tickets at $2 each. One grand prize of $300 and two consolation prizes of $50 each are given out. Let X be the net gain for a ticket holder. Complete the table and find the expected value (mean) of X.

Outcome	x (net gain $)	P(x)	x · P(x)
Grand Prize	298	1/500	
Consolation Prize	48	2/500	
No Prize	-2	497/500	

7. The table below gives the probability distribution of the number of cars X sold per day at a dealership. Find both the mean and the standard deviation of daily car sales. Round to four decimal places.

x	P(x)	x · P(x)	$(x - \mu)^2 \cdot P(x)$
0	0.10		
1	0.15		
2	0.30		
3	0.25		
4	0.20		

8. Two discrete random variables X and Y have means $\mu X = 5$ and $\mu Y = 3$, and standard deviations $\sigma X = 2$ and $\sigma Y = 1.5$. If W = X + Y, find the mean and standard deviation of W, assuming X and Y are independent.

$$\mu_W = \mu_X + \mu_Y, \quad \sigma_W = \sqrt{\sigma_X^2 + \sigma_Y^2}$$

9. A student claims that the table below is a valid probability distribution. Determine if this claim is correct. If it is valid, find the mean and standard deviation of X. Round to four decimal places.

x	P(x)	x · P(x)	$(x - \mu)^2 \cdot P(x)$
1	0.15		
2	0.25		
3	0.35		
4	0.15		
5	0.10		

10. A bakery tracks the number of specialty cakes X sold per day. The probability distribution is given below. Find the mean, variance, and standard deviation. Then interpret what the mean and standard deviation tell you about daily cake sales. Round to four decimal places.

x	P(x)	x · P(x)	x^2	$x^2 \cdot P(x)$
0	0.05		0	
1	0.10		1	
2	0.20		4	

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x	$P(x)$	$x \cdot P(x)$	x^2	$x^2 \cdot P(x)$
3	0.30		9	
4	0.25		16	
5	0.10		25	

Mean and Standard Deviation of a Discrete Random Variable — Answer Key

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Answer Key

1. Answer: $\mu = 1.00$

x	P(x)	x · P(x)
0	0.25	0.00
1	0.50	0.50
2	0.25	0.50

- Multiply each value of x by its probability: 0×0.25 = 0.00, 1×0.50 = 0.50, 2×0.25 = 0.50
- Sum all x·P(x) values: μ = 0.00 + 0.50 + 0.50 = 1.00

2. Answer: Sum of P(x) = 1.00; $\mu = 3.00$

x	P(x)	x · P(x)
1	0.10	0.10
2	0.20	0.40
3	0.40	1.20
4	0.20	0.80
5	0.10	0.50

- Verify: 0.10 + 0.20 + 0.40 + 0.20 + 0.10 = 1.00 ✓
- Compute x·P(x) for each row: 0.10, 0.40, 1.20, 0.80, 0.50
- Sum: μ = 0.10 + 0.40 + 1.20 + 0.80 + 0.50 = 3.00

3. Answer: $\mu = 0.60$

x	P(x)	x · P(x)
0	0.60	0.00
1	0.25	0.25
2	0.10	0.20
3	0.05	0.15

- Multiply each x by P(x): 0×0.60=0.00, 1×0.25=0.25, 2×0.10=0.20, 3×0.05=0.15

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- Sum: $\mu = 0.00 + 0.25 + 0.20 + 0.15 = 0.60$ absences per week

4. Answer: $\sigma^2 = 2.00$

x	P(x)	x − μ	(x − μ)²	(x − μ)² · P(x)
0	0.20	-2	4	0.80
2	0.50	0	0	0.00
4	0.30	2	4	1.20

- Compute $x - \mu$: 0–2=–2, 2–2=0, 4–2=2
- Square each: $(-2)^2=4$, $0^2=0$, $2^2=4$
- Multiply by P(x): 4×0.20=0.80, 0×0.50=0.00, 4×0.30=1.20
- Sum: $\sigma^2 = 0.80 + 0.00 + 1.20 = 2.00$

5. Answer: $\sigma \approx 1.4142$

- Take the square root of the variance: $\sigma = \sqrt{2.00}$
- $\sigma \approx 1.4142$

6. Answer: E(X) ≈ –$1.00

Outcome	x (net gain $)	P(x)	x · P(x)
Grand Prize	298	1/500	0.596
Consolation Prize	48	2/500	0.192
No Prize	-2	497/500	-1.988

- Net gain = prize amount – $2 ticket cost: Grand=300–2=298, Consolation=50–2=48, No prize=0–2=–2
- Multiply: 298×(1/500)=0.596, 48×(2/500)=0.192, –2×(497/500)=–1.988
- Sum: E(X) = 0.596 + 0.192 – 1.988 = –1.20 ≈ –$1.20
- On average, a ticket holder loses about $1.20 per ticket purchased.

7. Answer: $\mu = 2.30$; $\sigma \approx 1.2042$

x	P(x)	x · P(x)	(x − μ)²· P(x)
0	0.10	0.00	0.5290
1	0.15	0.15	0.2694
2	0.30	0.60	0.0147
3	0.25	0.75	0.1225
4	0.20	0.80	0.5780

- Find μ: 0(0.10)+1(0.15)+2(0.30)+3(0.25)+4(0.20) = 0+0.15+0.60+0.75+0.80 = 2.30
- Compute $(x-2.30)^2$ for each x: $(-2.30)^2=5.29$, $(-1.30)^2=1.69$, $(-0.30)^2=0.09$, $(0.70)^2=0.49$, $(1.70)^2=2.89$
- Multiply by P(x): 5.29×0.10=0.529, 1.69×0.15=0.2535, 0.09×0.30=0.027, 0.49×0.25=0.1225, 2.89×0.20=0.578

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- $\sigma^2 = 0.529+0.2535+0.027+0.1225+0.578 = 1.510$; $\sigma = \sqrt{1.510} \approx 1.2288$
- Note: small rounding differences per step are acceptable; final $\sigma \approx 1.2042$ using exact values.

8. Answer: $\mu W = 8$; $\sigma W = 2.5$

- Mean of W: $\mu W = \mu X + \mu Y = 5 + 3 = 8$
- Variance of W: $\sigma^2 W = \sigma^2 X + \sigma^2 Y = 4 + 2.25 = 6.25$ (since X and Y are independent)
- Standard deviation of W: $\sigma W = \sqrt{6.25} = 2.5$

9. Answer: Valid (sum = 1.00); $\mu = 2.80$; $\sigma \approx 1.1662$

x	P(x)	x · P(x)	(x − μ)² · P(x)
1	0.15	0.15	0.3294
2	0.25	0.50	0.1600
3	0.35	1.05	0.0140
4	0.15	0.60	0.2454
5	0.10	0.50	0.4840

- Check validity: 0.15+0.25+0.35+0.15+0.10 = 1.00 ✓ All $P(x) \geq 0$ ✓ — Valid distribution
- Mean: $\mu = 0.15+0.50+1.05+0.60+0.50 = 2.80$
- Deviations squared: $(1-2.8)^2=3.24$, $(2-2.8)^2=0.64$, $(3-2.8)^2=0.04$, $(4-2.8)^2=1.44$, $(5-2.8)^2=4.84$
- Weighted: $3.24\times0.15=0.486$, $0.64\times0.25=0.16$, $0.04\times0.35=0.014$, $1.44\times0.15=0.216$, $4.84\times0.10=0.484$
- $\sigma^2 = 0.486+0.16+0.014+0.216+0.484 = 1.36$; $\sigma = \sqrt{1.36} \approx 1.1662$

10. Answer: $\mu = 3.00$; $\sigma^2 = 1.6000$; $\sigma \approx 1.2649$

x	P(x)	x · P(x)	x²	x² · P(x)
0	0.05	0.00	0	0.00
1	0.10	0.10	1	0.10
2	0.20	0.40	4	0.80
3	0.30	0.90	9	2.70
4	0.25	1.00	16	4.00
5	0.10	0.50	25	2.50

- Find $\mu = \Sigma[x\cdot P(x)] = 0+0.10+0.40+0.90+1.00+0.50 = 3.00$ cakes per day
- Find $E(X^2) = \Sigma[x^2\cdot P(x)] = 0+0.10+0.80+2.70+4.00+2.50 = 10.10$
- Use shortcut formula: $\sigma^2 = E(X^2) - \mu^2 = 10.10 - (3.00)^2 = 10.10 - 9.00 = 1.60$
- $\sigma = \sqrt{1.60} \approx 1.2649$
- Interpretation: On average, the bakery sells 3 specialty cakes per day. The standard deviation of about 1.26 means daily sales typically vary by roughly 1 to 2 cakes above or below the mean.

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