Mean, Variance & Standard Deviation of a Binomial Distribution

Statistics Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Calculate the mean (expected value) of a binomial distribution using μ = np
- Calculate the variance of a binomial distribution using σ² = npq
- Calculate the standard deviation of a binomial distribution using σ = √(npq)

Problems

1. A fair coin is flipped 10 times. Find the mean (expected value) of the number of heads.

$$n = 10, \quad p = 0.5, \quad \mu = np$$

2. A die is rolled 12 times. Success is defined as rolling a 3. Find the expected number of 3s.

$$n = 12, \quad p = \frac{1}{6}, \quad \mu = np$$

3. A student randomly guesses on a 20-question true-or-false quiz. Find the variance of the number of correct answers.

$$n = 20, \quad p = 0.5, \quad q = 0.5, \quad \sigma^2 = npq$$

4. A basketball player makes 70% of her free throw shots. She attempts 15 shots in a game. Find the standard deviation of the number of made shots.

$$n = 15, \quad p = 0.70, \quad q = 0.30, \quad \sigma = \sqrt{npq}$$

5. A call center agent successfully resolves 80% of all customer calls. If 50 calls come in during a shift, find both the mean and variance of the number of successfully resolved calls.

$$n = 50, \quad p = 0.80, \quad q = 0.20$$

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6. A factory produces light bulbs, and 5% of them are defective. A quality inspector randomly selects 200 bulbs. Find the expected number of defective bulbs and the standard deviation.

$$n = 200, \quad p = 0.05, \quad q = 0.95$$

7. A survey shows that 65% of adults in a city own a smartphone. A researcher surveys 300 adults. Find the mean, variance, and standard deviation of the number of smartphone owners.

$$n = 300, \quad p = 0.65, \quad q = 0.35$$

8. A new drug is effective 88% of the time. A clinic administers the drug to 500 patients. Complete the table below by finding the missing values for mean, variance, and standard deviation.

n	p	q	Mean (μ)	Variance (σ²)	Std Dev (σ)
500	0.88	0.12			

9. The table below shows four different binomial scenarios. Fill in all missing values for mean, variance, and standard deviation for each scenario.

Scenario	n	p	q	Mean (μ)	Variance (σ²)	Std Dev (σ)
A	25	0.40	0.60			
B	100	0.75	0.25			
C	60	0.30	0.70			
D	400	0.55	0.45			

10. A renowned archer hits the bullseye 94% of the time. She is preparing for a competition where she will shoot 750 arrows. Use the binomial distribution to find the expected number of bullseyes, the variance, and the standard deviation. Then interpret what the standard deviation means in context.

$n = 750, \quad p = 0.94, \quad q = 0.06, \quad \mu = np, \quad \sigma^2 = npq, \quad \sigma = \sqrt{npq}$

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Mean, Variance & Standard Deviation of a Binomial Distribution — Answer Key

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Answer Key

1. Answer: $\mu = 5$
- Identify n = 10 and p = 0.5
- Apply the formula: $\mu = np = 10 \times 0.5 = 5$

2. Answer: $\mu = 2$
- Identify n = 12 and p = 1/6
- Apply the formula: $\mu = np = 12 \times (1/6) = 2$

3. Answer: $\sigma^2 = 5$
- Identify n = 20, p = 0.5, q = 1 – 0.5 = 0.5
- Apply the formula: $\sigma^2 = npq = 20 \times 0.5 \times 0.5 = 5$

4. Answer: $\sigma \approx 1.77$
- Identify n = 15, p = 0.70, q = 0.30
- Calculate variance: $\sigma^2 = npq = 15 \times 0.70 \times 0.30 = 3.15$
- Take the square root: $\sigma = \sqrt{3.15} \approx 1.77$

5. Answer: $\mu = 40$, $\sigma^2 = 8$
- Identify n = 50, p = 0.80, q = 0.20
- Mean: $\mu = np = 50 \times 0.80 = 40$
- Variance: $\sigma^2 = npq = 50 \times 0.80 \times 0.20 = 8$

6. Answer: $\mu = 10$, $\sigma \approx 3.08$
- Identify n = 200, p = 0.05, q = 0.95
- Expected value (mean): $\mu = np = 200 \times 0.05 = 10$
- Variance: $\sigma^2 = npq = 200 \times 0.05 \times 0.95 = 9.5$
- Standard deviation: $\sigma = \sqrt{9.5} \approx 3.08$

7. Answer: $\mu = 195$, $\sigma^2 = 68.25$, $\sigma \approx 8.26$
- Identify n = 300, p = 0.65, q = 0.35
- Mean: $\mu = np = 300 \times 0.65 = 195$
- Variance: $\sigma^2 = npq = 300 \times 0.65 \times 0.35 = 68.25$
- Standard deviation: $\sigma = \sqrt{68.25} \approx 8.26$

8. Answer: $\mu = 440$, $\sigma^2 = 52.8$, $\sigma \approx 7.27$

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n	p	q	Mean (μ)	Variance (σ²)	Std Dev (σ)
500	0.88	0.12	440	52.8	7.27

- Identify n = 500, p = 0.88, q = 0.12
- Mean: μ = np = 500 × 0.88 = 440
- Variance: σ² = npq = 500 × 0.88 × 0.12 = 52.8
- Standard deviation: σ = √52.8 ≈ 7.27

9. Answer: A: μ=10, σ²=6, σ≈2.45 | B: μ=75, σ²=18.75, σ≈4.33 | C: μ=18, σ²=12.6, σ≈3.55 | D: μ=220, σ²=99, σ≈9.95

Scenario	n	p	q	Mean (μ)	Variance (σ²)	Std Dev (σ)
A	25	0.40	0.60	10	6	2.45
B	100	0.75	0.25	75	18.75	4.33
C	60	0.30	0.70	18	12.6	3.55
D	400	0.55	0.45	220	99	9.95

- Scenario A: μ = 25×0.40 = 10; σ² = 25×0.40×0.60 = 6; σ = √6 ≈ 2.45
- Scenario B: μ = 100×0.75 = 75; σ² = 100×0.75×0.25 = 18.75; σ = √18.75 ≈ 4.33
- Scenario C: μ = 60×0.30 = 18; σ² = 60×0.30×0.70 = 12.6; σ = √12.6 ≈ 3.55
- Scenario D: μ = 400×0.55 = 220; σ² = 400×0.55×0.45 = 99; σ = √99 ≈ 9.95

10. Answer: μ = 705, σ² = 42.3, σ ≈ 6.50; on average she hits 705 bullseyes, typically varying by about 6.50 bullseyes

- Identify n = 750, p = 0.94, q = 1 – 0.94 = 0.06
- Expected value (mean): μ = np = 750 × 0.94 = 705
- Variance: σ² = npq = 750 × 0.94 × 0.06 = 42.3
- Standard deviation: σ = √42.3 ≈ 6.50
- Interpretation: The archer is expected to hit 705 bullseyes out of 750 shots, with scores typically varying by about 6.50 bullseyes from this expected value.

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