Probability, Expected Value & Chi-Square Goodness-of-Fit

AP Statistics Worksheet · Grade 11–12

Name: ______________________________ Date: _________________

Learning Objectives

- Calculate probabilities for independent repeated events using multiplication rules
- Construct probability models and compute expected values for discrete random variables
- Conduct a chi-square goodness-of-fit test including hypotheses, conditions, test statistic, and conclusion

Problems

1. A spinner has four equally likely outcomes: Skunk, \$100, \$200, and \$500. What is the probability of landing on \$500 on a single spin?

$$P(\$500) = ?$$

2. Using the same spinner with four equally likely outcomes (Skunk, \$100, \$200, \$500), what is the probability of NOT landing on a Skunk on a single spin?

$$P(\text{not Skunk}) = ?$$

3. A contestant spins the wheel (four equally likely outcomes: Skunk, \$100, \$200, \$500) twice. Assuming independence, what is the probability of winning money (not Skunk) on both spins?

$$P(\text{not Skunk on spin 1 AND spin 2}) = \left(\frac{3}{4}\right)^2$$

4. From the 2003 AP Stats problem: the spinner has four equally likely outcomes (Skunk, \$100, \$200, \$500). Find the probability of winning money (not Skunk) on all three of the first three spins.

$$P(\text{number on all 3 spins}) = \left(\frac{3}{4}\right)^3$$

Scan to watch

5. A contestant has already earned $800 after three spins. They are deciding whether to take a fourth spin. Complete the probability model table below showing the total winnings and their probabilities if they spin again. Each of the four outcomes is equally likely.

Outcome	Added Amount	Total Winnings	Probability
Skunk	$0 (lose all)		0.25
$100	$100		0.25
$200	$200		0.25
$500	$500		0.25

6. Using the probability model from the previous problem, compute the expected total winnings if the contestant (who has $800) takes a fourth spin. Should the contestant spin again if their goal is to maximize expected winnings?

$$E(X) = \sum x_i \cdot P(x_i) = 0(0.25) + 900(0.25) + 1000(0.25) + 1300(0.25)$$

7. In 100 simulated spins of the game wheel, the observed frequencies were: Skunk = 32, $100 = 21, $200 = 28, $500 = 19. If the wheel is fair, what are the expected frequencies for each outcome in 100 spins?

Outcome	Observed Frequency	Expected Frequency
Skunk	32	
$100	21	
$200	28	
$500	19	

8. State the null and alternative hypotheses for a chi-square goodness-of-fit test to determine whether the game wheel is fair. Let p1, p2, p3, and p4 represent the probabilities of Skunk, $100, $200, and $500 respectively.

H_0: $p_1 = p_2 = p_3 = p_4 = 0.25$ (wheel is fair)

H_1: At least one $p_i \neq 0.25$ (wheel is not fair)

Scan to watch

9. Using the observed frequencies (Skunk = 32, \$100 = 21, \$200 = 28, \$500 = 19) and expected frequency of 25 for each outcome in 100 spins, calculate the chi-square test statistic. Then check the normality condition and find the degrees of freedom.

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

10. Using the chi-square test statistic of approximately 4.40 with 3 degrees of freedom, the critical value at significance level α = 0.05 is 7.815. State your conclusion about whether the game wheel is fair, and interpret the result in context. Also explain what a Type I error would mean in this context.

H_0: $p_1 = p_2 = p_3 = p_4 = 0.25$

H_1: At least one $p_i \neq 0.25$

$$\chi^2_{\text{critical}} = 7.815 \quad (\alpha = 0.05,\ df = 3)$$

Scan to watch

Probability, Expected Value & Chi-Square Goodness-of-Fit — Answer Key

AP Statistics Worksheet · Grade 11–12

Answer Key

1. Answer: 1/4 or 0.25

- There are 4 equally likely outcomes: Skunk, \$100, \$200, \$500.
- P(\$500) = 1/4 = 0.25

2. Answer: 3/4 or 0.75

- P(Skunk) = 1/4
- P(not Skunk) = 1 − 1/4 = 3/4 = 0.75

3. Answer: 9/16 or 0.5625

- P(not Skunk on one spin) = 3/4
- Since spins are independent: P(not Skunk twice) = (3/4) × (3/4) = 9/16 = 0.5625

4. Answer: 27/64 ≈ 0.4219

- P(not Skunk on one spin) = 3/4 = 0.75
- Spins are independent events.
- P(not Skunk on all 3 spins) = $(3/4)^3$ = 27/64 ≈ 0.4219

5. Answer: Total Winnings: \$0, \$900, \$1000, \$1300 each with probability 0.25

Outcome	Added Amount	Total Winnings	Probability
Skunk	\$0 (lose all)	\$0	0.25
\$100	\$100	\$900	0.25
\$200	\$200	\$1,000	0.25
\$500	\$500	\$1,300	0.25

- If Skunk: contestant loses all money → Total = \$0
- If \$100: \$800 + \$100 = \$900
- If \$200: \$800 + \$200 = \$1,000
- If \$500: \$800 + \$500 = \$1,300
- Each outcome has probability 1/4 = 0.25

6. Answer: E(X) = \$800; The expected value equals the current amount, so there is no financial gain from spinning.

- E(X) = 0(0.25) + 900(0.25) + 1000(0.25) + 1300(0.25)
- E(X) = 0 + 225 + 250 + 325

Scan to watch

- $E(X) = 800$
- The expected value equals the current winnings of $800.
- Conclusion: There is no expected gain from spinning; the decision depends on risk tolerance.

7. Answer: Each expected frequency = 25

Outcome	Observed Frequency	Expected Frequency
Skunk	32	25
$100	21	25
$200	28	25
$500	19	25

- Under a fair wheel, each of 4 outcomes has probability 1/4.
- Expected frequency = $100 \times (1/4) = 25$ for each outcome.
- All four expected frequencies are 25.

8. Answer: H_0: all probabilities equal 0.25; H_1: at least one probability differs from 0.25

- The null hypothesis states the wheel is fair: each outcome has probability 1/4 = 0.25.
- The alternative hypothesis states at least one outcome probability differs from 0.25.
- This is a chi-square goodness-of-fit test with $k = 4$ categories.

9. Answer: $\chi^2 = 4.24$, df = 3; normality condition met (all expected counts ≥ 5)

- $\chi^2 = (32-25)^2/25 + (21-25)^2/25 + (28-25)^2/25 + (19-25)^2/25$
- $\chi^2 = 49/25 + 16/25 + 9/25 + 36/25$
- $\chi^2 = 1.96 + 0.64 + 0.36 + 1.44 = 4.40$
- Degrees of freedom = $k - 1 = 4 - 1 = 3$
- Normality condition: all expected counts = 25 ≥ 5 ✓

10. Answer: Fail to reject H_0; $\chi^2 = 4.40 < 7.815$. Insufficient evidence the wheel is unfair. Type I error: concluding the wheel is unfair when it actually is fair.

- Test statistic: $\chi^2 \approx 4.40$
- Critical value at $\alpha = 0.05$, df = 3: $\chi^2\_critical = 7.815$
- Since $4.40 < 7.815$, we fail to reject H_0.
- Conclusion: There is insufficient statistical evidence at the 0.05 level to conclude the wheel is unfair.
- Type I error: Rejecting H_0 when it is actually true — concluding the wheel is biased when it is actually fair.
- This would occur with probability $\alpha = 0.05$.