Probability Using Discrete Random Variables

Statistics Worksheet · Grade 9–12

Name: _____ Date: _____

Learning Objectives

- Identify discrete and continuous random variables in real-world contexts
- Construct a probability model (probability distribution table) for a discrete random variable
- Calculate probabilities using a probability distribution of a discrete random variable

Problems

1. Classify each of the following as a discrete or continuous random variable. (a) The number of students absent in a class on a given day. (b) The height of a randomly selected student. (c) The number of heads when a coin is flipped 5 times. (d) The amount of rainfall in a city in one month.

2. A single fair die is rolled. Let X be the outcome showing on the top face. List all possible values of X and verify that this is a discrete random variable.

$$X \in \{1, 2, 3, 4, 5, 6\}$$

3. A fair coin is flipped once. Let X = 1 if the result is Heads and X = 0 if the result is Tails. Complete the probability distribution table below.

x	P(X = x)
0	
1	

4. A bag contains 3 red marbles and 2 blue marbles. One marble is drawn at random. Let X = 1 if the marble is red and X = 0 if the marble is blue. Build the probability distribution table and find P(X = 1).

x	P(X = x)
0	
1	

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5. Two fair coins are tossed simultaneously. Let X be the number of heads obtained. Construct the complete probability distribution table for X.

x	Outcomes	P(X = x)
0		
1		
2		

6. The table below shows the probability distribution of X, the number of goals scored by a soccer team in a game. Verify that this is a valid probability distribution, then find P(X is at least 2).

x	0	1	2	3	4
P(X = x)	0.10	0.25	0.35	0.20	0.10

7. A student randomly guesses on a 3-question true/false quiz. Let X be the number of correct answers. Construct the probability distribution table and find the probability that the student gets exactly 2 correct answers.

$$P(X = k) = \binom{3}{k}\left(\frac{1}{2}\right)^3$$

8. The probability distribution of X is given in the table. Find the missing probability value k, then calculate $P(1 \le X \le 3)$.

x	0	1	2	3	4
P(X = x)	0.05	0.20	k	0.30	0.15

9. A raffle sells 100 tickets. There is 1 prize of $500, 2 prizes of $100, and 5 prizes of $20. The rest win nothing. Let X be the prize amount won by a randomly selected ticket. Construct the probability distribution and calculate the expected value E(X) — that is, the average amount a ticket holder can expect to win.

$$E(X) = \sum x \cdot P(X = x)$$

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10. A discrete random variable X has the probability distribution shown. Determine whether the distribution is valid. If valid, find: (a) P(X > 2), (b) P(X is even), and (c) the expected value E(X). If not valid, explain why.

x	1	2	3	4	5
P(X = x)	0.15	0.25	0.30	0.20	0.10

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Probability Using Discrete Random Variables — Answer Key

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Answer Key

1. Answer: (a) Discrete, (b) Continuous, (c) Discrete, (d) Continuous

- A discrete random variable takes countable, separate values (whole numbers).
- A continuous random variable takes any value within an interval (measurable quantities).
- (a) Number of absences — countable whole numbers → Discrete.
- (b) Height — can be any value in a range → Continuous.
- (c) Number of heads — countable outcomes (0–5) → Discrete.
- (d) Rainfall amount — measured on a continuous scale → Continuous.

2. Answer: X can take values 1, 2, 3, 4, 5, 6 — a finite countable set, so X is discrete.

- Rolling a fair die produces one of six outcomes: 1, 2, 3, 4, 5, or 6.
- Each outcome is a distinct whole number — the set is finite and countable.
- Because X can only take specific separated values (not a continuous range), X is a discrete random variable.

3. Answer: P(X=0) = 1/2, P(X=1) = 1/2

x	P(X = x)
0	1/2
1	1/2

- A fair coin has two equally likely outcomes: Heads (H) and Tails (T).
- P(X = 0) = P(Tails) = 1/2.
- P(X = 1) = P(Heads) = 1/2.
- Check: 1/2 + 1/2 = 1 ✓ — all probabilities sum to 1.

4. Answer: P(X=0) = 2/5, P(X=1) = 3/5

x	P(X = x)
0	2/5
1	3/5

- Total marbles = 3 red + 2 blue = 5.
- P(X = 1) = P(red) = 3/5.
- P(X = 0) = P(blue) = 2/5.
- Verify: 3/5 + 2/5 = 5/5 = 1 ✓.

5. Answer: P(X=0)=1/4, P(X=1)=2/4=1/2, P(X=2)=1/4

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x	Outcomes	P(X = x)
0	TT	1/4
1	HT, TH	2/4 = 1/2
2	HH	1/4

- Sample space: {HH, HT, TH, TT} — 4 equally likely outcomes.
- X = 0 (no heads): only TT → P = 1/4.
- X = 1 (one head): HT or TH → P = 2/4 = 1/2.
- X = 2 (two heads): only HH → P = 1/4.
- Sum check: 1/4 + 2/4 + 1/4 = 4/4 = 1 ✓.

6. Answer: Valid distribution. P(X ≥ 2) = 0.65

- Check validity: 0.10 + 0.25 + 0.35 + 0.20 + 0.10 = 1.00 ✓.
- All probabilities are between 0 and 1 ✓ — valid distribution.
- P(X ≥ 2) = P(X=2) + P(X=3) + P(X=4) = 0.35 + 0.20 + 0.10 = 0.65.

7. Answer: P(X = 2) = 3/8

- Each question has probability 1/2 of being correct; questions are independent.
- Using the binomial formula with n=3, p=1/2:
- P(X=0) = C(3,0)(1/2)^3 = 1/8.
- P(X=1) = C(3,1)(1/2)^3 = 3/8.
- P(X=2) = C(3,2)(1/2)^3 = 3/8.
- P(X=3) = C(3,3)(1/2)^3 = 1/8.
- Sum = 1/8 + 3/8 + 3/8 + 1/8 = 8/8 = 1 ✓.
- Answer: P(X = 2) = 3/8 = 0.375.

8. Answer: k = 0.30; P(1 ≤ X ≤ 3) = 0.80

- The sum of all probabilities must equal 1.
- 0.05 + 0.20 + k + 0.30 + 0.15 = 1.
- 0.70 + k = 1 → k = 0.30.
- P(1 ≤ X ≤ 3) = P(X=1) + P(X=2) + P(X=3) = 0.20 + 0.30 + 0.30 = 0.80.

9. Answer: E(X) = $8.00

- Identify possible values: X ∈ {0, 20, 100, 500}.
- P(X = 500) = 1/100 = 0.01.
- P(X = 100) = 2/100 = 0.02.
- P(X = 20) = 5/100 = 0.05.
- P(X = 0) = 92/100 = 0.92.
- Check: 0.01 + 0.02 + 0.05 + 0.92 = 1.00 ✓.
- E(X) = 500(0.01) + 100(0.02) + 20(0.05) + 0(0.92)
- E(X) = 5 + 2 + 1 + 0 = $8.00.

10. Answer: Valid. (a) P(X>2) = 0.60, (b) P(X even) = 0.45, (c) E(X) = 2.85

- Check validity: 0.15 + 0.25 + 0.30 + 0.20 + 0.10 = 1.00 ✓. All values in [0,1] ✓.

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- (a) P(X > 2) = P(X=3) + P(X=4) + P(X=5) = 0.30 + 0.20 + 0.10 = 0.60.
- (b) Even values of X: 2 and 4. P(X even) = P(X=2) + P(X=4) = 0.25 + 0.20 = 0.45.
- (c) E(X) = 1(0.15) + 2(0.25) + 3(0.30) + 4(0.20) + 5(0.10)
- E(X) = 0.15 + 0.50 + 0.90 + 0.80 + 0.50 = 2.85.

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