Sampling Distribution: Mean & Standard Deviation

Statistics Worksheet · Grade 10–12

Name: ________________________________ Date: __________________

Learning Objectives

- Apply the Central Limit Theorem to identify the mean of a sampling distribution
- Calculate the standard deviation (standard error) of a sampling distribution using σ/√n
- Interpret how increasing sample size affects the sampling distribution's standard deviation

Problems

1. A population has a mean of 50 and a standard deviation of 10. What is the mean of the sampling distribution of the sample mean?

$$\mu_{\bar{x}} = ?$$

2. A population has a standard deviation of 12. If a random sample of size 9 is selected, what is the standard deviation of the sampling distribution (standard error)?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

3. The heights of young women are normally distributed with a mean of 64.5 inches and a standard deviation of 2.5 inches. Find the mean and standard deviation of the sampling distribution when 10 women are randomly selected.

$$\mu_{\bar{x}} = \mu, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

4. Using the same women's height distribution (mean = 64.5 inches, standard deviation = 2.5 inches), find the standard deviation of the sampling distribution when 100 women are randomly selected. How does this compare to the result when n = 10?

$$\sigma_{\bar{x}} = \frac{2.5}{\sqrt{100}}$$

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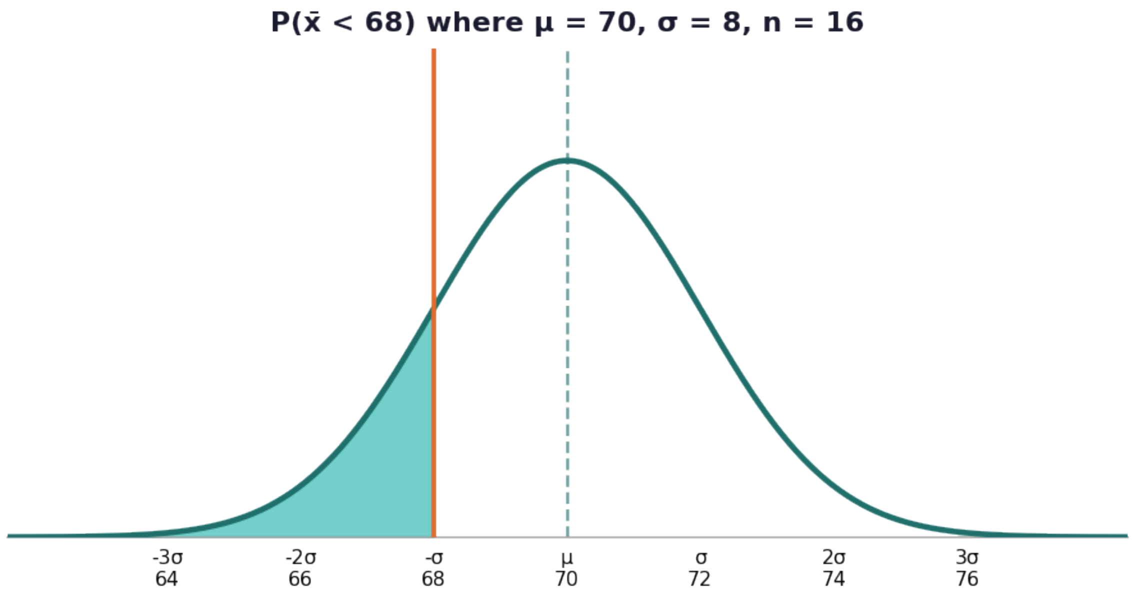
5. Complete the table below by calculating the standard deviation of the sampling distribution for each given sample size. The population standard deviation is 15.

Sample Size (n)	σ	σ_x̄ = σ / √n
25	15	
100	15	
225	15	
400	15	

6. The weights of adult male dogs in a breed are normally distributed with a mean of 70 pounds and a standard deviation of 8 pounds. What is the probability that the mean weight of a random sample of 16 dogs is less than 68 pounds?



7. Exam scores at a university are normally distributed with a mean of 75 and a standard deviation of 10. A professor randomly selects 25 students. What is the probability that their mean score is greater than 78?

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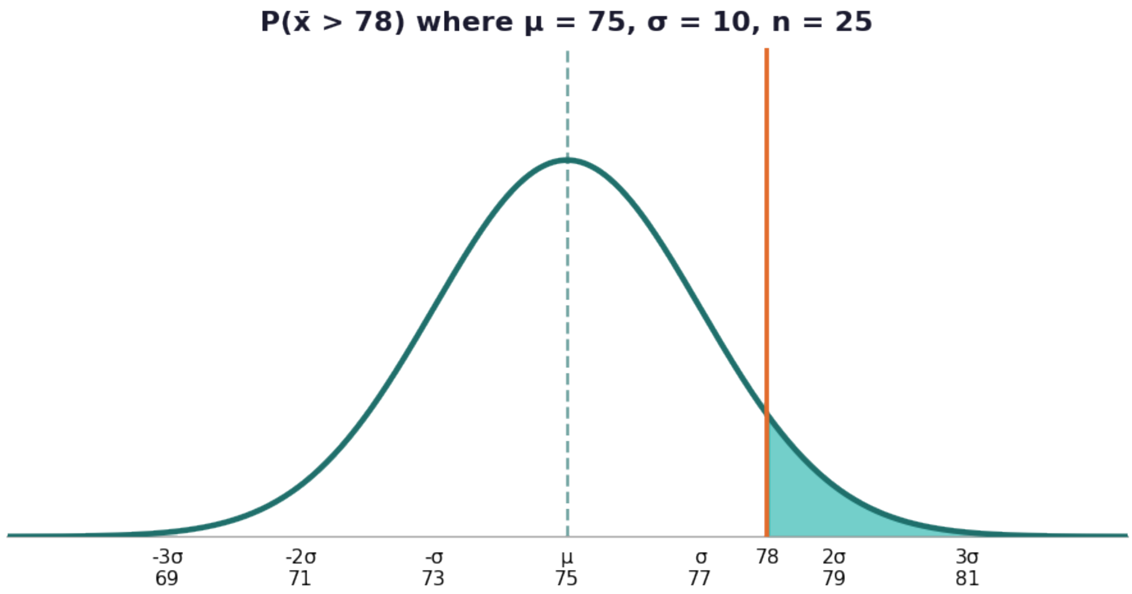
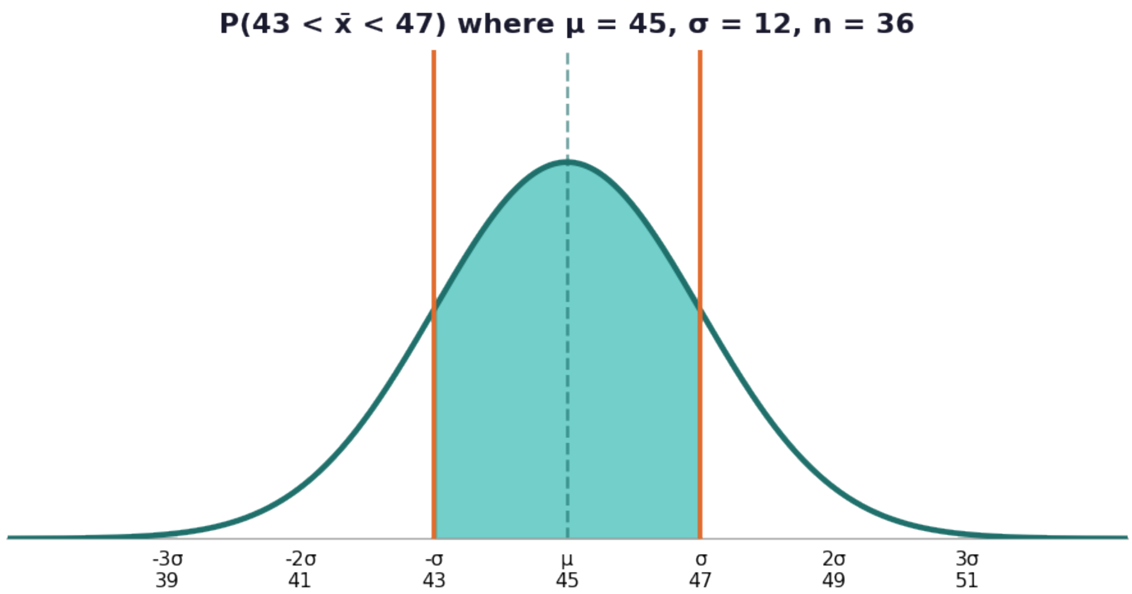


8. The daily commute time (in minutes) for workers in a city is normally distributed with a mean of 45 minutes and a standard deviation of 12 minutes. What is the probability that the mean commute time for a random sample of 36 workers is between 43 and 47 minutes?



9. A soft drink machine dispenses drinks with a mean volume of 355 mL and a standard deviation of 5 mL. A quality control inspector checks a sample of 40 drinks. What sample size would be needed to reduce the standard error to exactly 0.5 mL? Show your work using the standard error formula.

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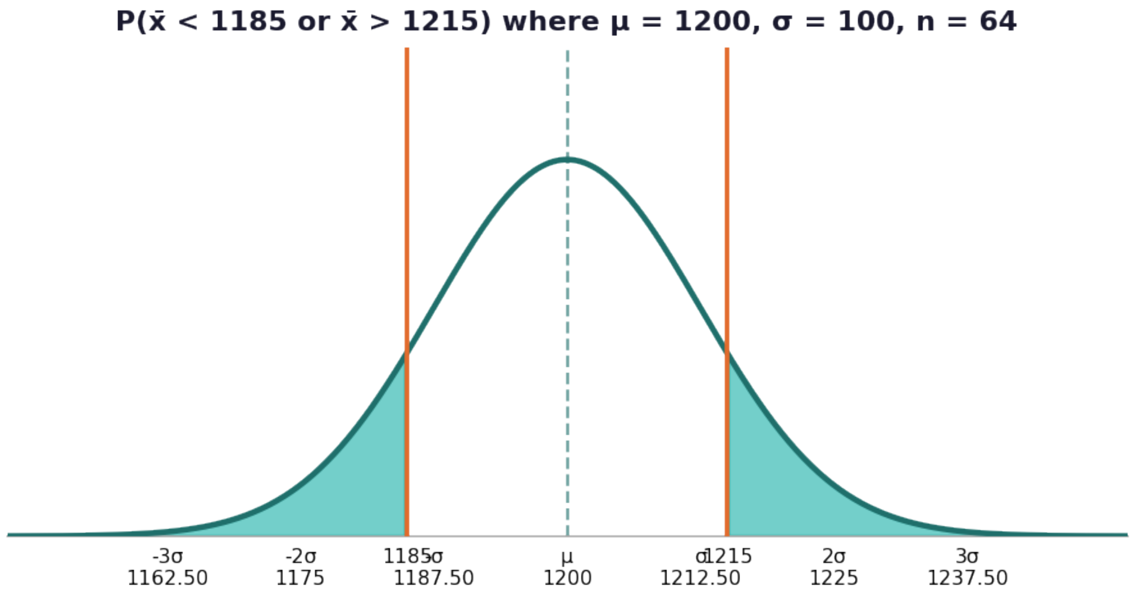
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 0.5$$

10. The lifespans of a brand of light bulbs are normally distributed with a mean of 1200 hours and a standard deviation of 100 hours. A retailer buys a shipment and randomly tests 64 bulbs. What is the probability that the sample mean lifespan falls outside the range of 1185 to 1215 hours? (Hint: find the probability of both tails combined.)



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Sampling Distribution: Mean & Standard Deviation — Answer Key

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Answer Key

1. Answer: $\mu_{\bar{x}} = 50$

- By the Central Limit Theorem, the mean of the sampling distribution equals the population mean.
- $\mu_{\bar{x}} = \mu = 50$

2. Answer: $\sigma_{\bar{x}} = 4$

- Use the formula: $\sigma_{\bar{x}} = \sigma / \sqrt{n}$
- $\sigma_{\bar{x}} = 12 / \sqrt{9} = 12 / 3 = 4$

3. Answer: $\mu_{\bar{x}} = 64.5$ in, $\sigma_{\bar{x}} \approx 0.79$ in

- Mean of sampling distribution: $\mu_{\bar{x}} = \mu = 64.5$ inches
- Standard deviation: $\sigma_{\bar{x}} = 2.5 / \sqrt{10} = 2.5 / 3.162 \approx 0.79$ inches

4. Answer: $\sigma_{\bar{x}} = 0.25$ in; smaller than when n = 10 (0.79 in)

- $\sigma_{\bar{x}} = 2.5 / \sqrt{100} = 2.5 / 10 = 0.25$ inches
- When $n = 10$, $\sigma_{\bar{x}} \approx 0.79$; when $n = 100$, $\sigma_{\bar{x}} = 0.25$ — a larger sample size produces a smaller standard error.

5. Answer: See completed table

Sample Size (n)	σ	$\sigma_{\bar{x}} = \sigma / \sqrt{n}$
25	15	3.00
100	15	1.50
225	15	1.00
400	15	0.75

- n=25: $15/\sqrt{25} = 15/5 = 3.00$
- n=100: $15/\sqrt{100} = 15/10 = 1.50$
- n=225: $15/\sqrt{225} = 15/15 = 1.00$
- n=400: $15/\sqrt{400} = 15/20 = 0.75$

6. Answer: $P \approx 0.1587$ (15.87%)

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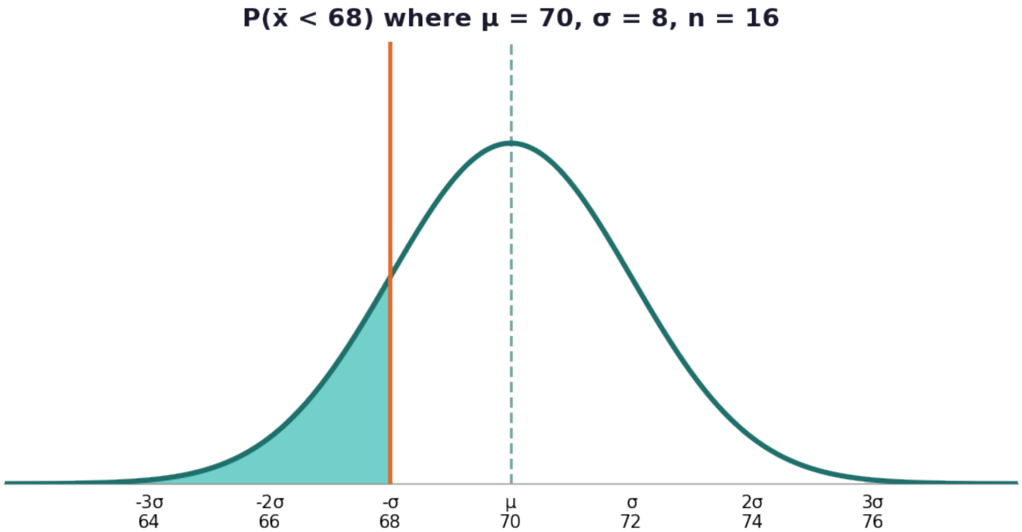
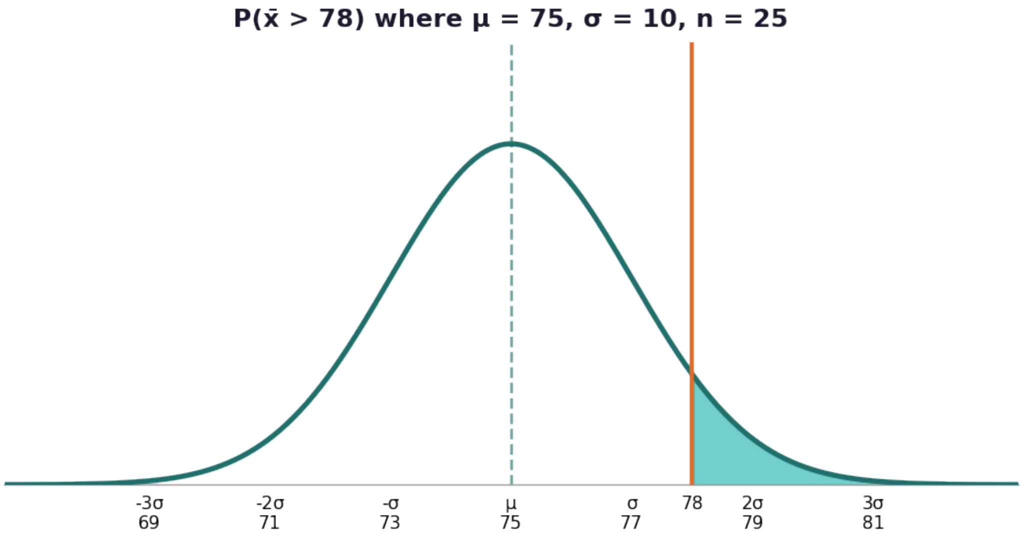


- First find the standard error: σ_x■ = 8 / √16 = 8 / 4 = 2
- Compute z: z = (68 – 70) / 2 = –2 / 2 = –1.00
- Look up P(Z < –1.00) in the standard normal table: P ≈ 0.1587

7. Answer: P ≈ 0.0668 (6.68%)



- Standard error: σ_x■ = 10 / √25 = 10 / 5 = 2
- z = (78 – 75) / 2 = 3 / 2 = 1.50
- P(Z > 1.50) = 1 – P(Z < 1.50) = 1 – 0.9332 = 0.0668

8. Answer: P ≈ 0.6827 (68.27%)

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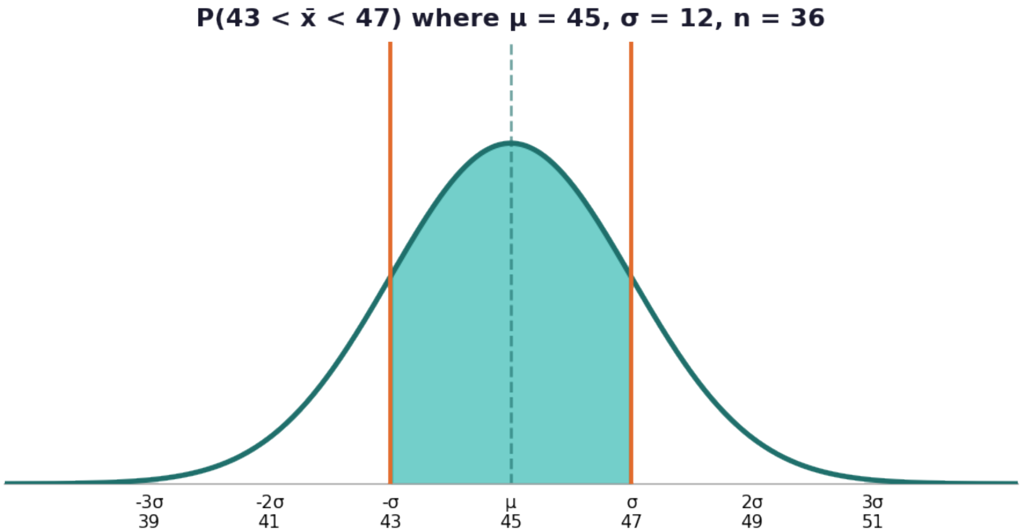

- Standard error: σ_x■ = 12 / √36 = 12 / 6 = 2
- z■ = (43 – 45) / 2 = –1.00 and z■ = (47 – 45) / 2 = 1.00
- P(–1 < Z < 1) = P(Z < 1) – P(Z < –1) = 0.8413 – 0.1587 = 0.6827

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9. Answer: n = 100

- Set the formula equal to the desired standard error: 5 / √n = 0.5
- Solve for √n: √n = 5 / 0.5 = 10
- Square both sides: $n = 10^2 = 100$

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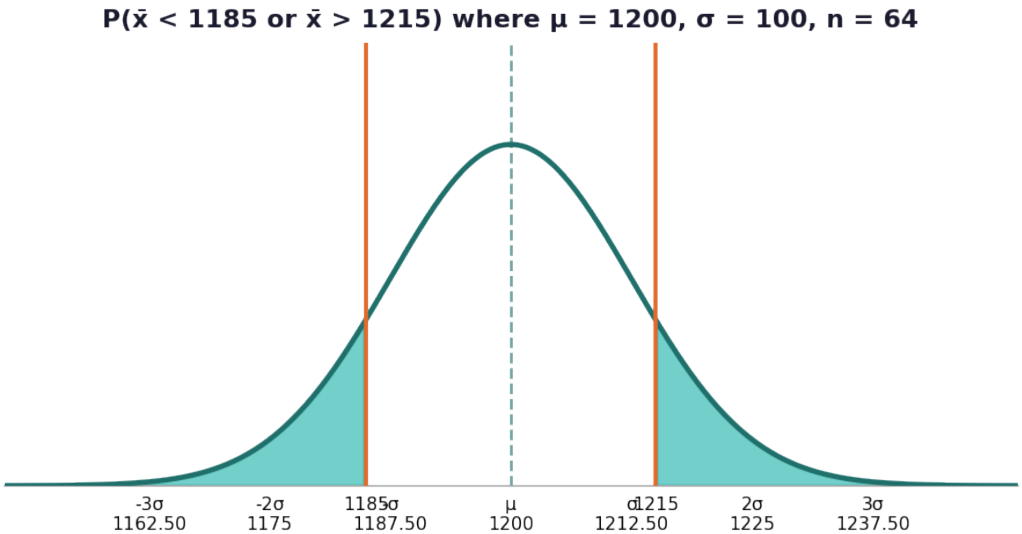
10. Answer: P ≈ 0.2302 (23.02%)



- Standard error: σ_x■ = 100 / √64 = 100 / 8 = 12.5
- z■ = (1185 – 1200) / 12.5 = –15 / 12.5 = –1.20 and z■ = (1215 – 1200) / 12.5 = 1.20
- P(Z < –1.20) = 0.1151 and P(Z > 1.20) = 0.1151
- Total probability = 0.1151 + 0.1151 = 0.2302

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