Six Trigonometric Ratios & the Pythagorean Theorem

Trigonometry Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Use the Pythagorean Theorem to find a missing side of a right triangle before applying trig ratios
- Evaluate all six trigonometric ratios (sin, cos, tan, csc, sec, cot) for a given angle in a right triangle
- Convert between degrees and radians and evaluate the six trig functions for special angles using the unit circle

Problems

1. A right triangle has legs of length 3 and 4. Find the hypotenuse using the Pythagorean Theorem.

$$c^2 = 3^2 + 4^2$$

2. A right triangle has a hypotenuse of 10 and one leg of 6. Find the missing leg using the Pythagorean Theorem.

$$10^2 = 6^2 + x^2$$

3. Using the triangle with hypotenuse 2, one leg 1, and the other leg equal to the square root of 3, find sin 30°.

$$\sin 30^\circ = \frac{\text{opposite}}{\text{hypotenuse}}$$

4. Using the same 30-60-90 triangle (sides 1, square root of 3, and 2), find cos 30° and tan 30°. Rationalize tan 30°.

$$\cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

5. Find the three reciprocal trig ratios (csc, sec, and cot) for 30° using the 30-60-90 triangle.

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$$\csc 30^\circ = \frac{1}{\sin 30^\circ}, \quad \sec 30^\circ = \frac{1}{\cos 30^\circ}, \quad \cot 30^\circ = \frac{1}{\tan 30^\circ}$$

6. Convert the angle pi over 3 radians into degrees, then state the coordinates of that angle on the unit circle.

$$\frac{\pi}{3} \times \frac{180^\circ}{\pi}$$

7. Using the unit circle coordinates for pi over 3, evaluate all six trig functions for theta equals pi over 3.

$$\theta = \frac{\pi}{3}, \quad (x, y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

8. A right triangle has a hypotenuse of 5 and one leg of 2. First find the missing leg, then evaluate all six trig ratios for the acute angle opposite the leg of length 2. Rationalize all answers.

$$c = 5, \quad a = 2, \quad b = ?$$

9. Given that sin(theta) equals 5 over 13 and theta is in the first quadrant, use the Pythagorean identity to find cos(theta), then evaluate all six trig functions.

$$\sin^2\theta + \cos^2\theta = 1, \quad \sin\theta = \frac{5}{13}$$

10. A right triangle is placed in the coordinate plane with angle theta at the origin. The terminal side passes through the point (negative 3, 4). Find all six trig functions of theta, rationalizing where necessary.

$$r = \sqrt{(-3)^2 + 4^2}$$

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Six Trigonometric Ratios & the Pythagorean Theorem — Answer Key

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Answer Key

1. Answer: c = 5
- $c^2 = 9 + 16 = 25$
- $c = \sqrt{25} = 5$

2. Answer: x = 8
- $100 = 36 + x^2$
- $x^2 = 64$, so $x = 8$

3. Answer: sin 30° = 1/2
- The side opposite 30° is 1, and the hypotenuse is 2
- sin 30° = 1/2

4. Answer: cos 30° = √3/2 ; tan 30° = √3/3
- cos 30° = adjacent/hypotenuse = √3/2
- tan 30° = opposite/adjacent = 1/√3 = √3/3 after rationalizing

5. Answer: csc 30° = 2 ; sec 30° = 2√3/3 ; cot 30° = √3
- csc 30° = 1/(1/2) = 2
- sec 30° = 1/(√3/2) = 2/√3 = 2√3/3 after rationalizing; cot 30° = 1/(√3/3) = √3

6. Answer: 60° ; coordinates (1/2, √3/2)
- (π/3) × (180°/π) = 60°
- On the unit circle, 60° has coordinates (cos 60°, sin 60°) = (1/2, √3/2)

7. Answer: sin=√3/2, cos=1/2, tan=√3, csc=2√3/3, sec=2, cot=√3/3
- sin(π/3)=y=√3/2; cos(π/3)=x=1/2; tan(π/3)=y/x=√3
- csc(π/3)=1/y=2/√3=2√3/3; sec(π/3)=1/x=2; cot(π/3)=x/y=1/√3=√3/3

8. Answer: b=√21; sin=2/5, cos=√21/5, tan=2/√21=2√21/21, csc=5/2, sec=5/√21=5√21/21, cot=√21/2
- $b^2 = 5^2 – 2^2 = 25 – 4 = 21$, so $b = \sqrt{21}$
- sin=2/5; cos=√21/5; tan=2/√21=2√21/21; csc=5/2; sec=5/√21=5√21/21; cot=√21/2

9. Answer: cos=12/13, tan=5/12, csc=13/5, sec=13/12, cot=12/5
- $\cos^2\theta = 1 – (5/13)^2 = 1 – 25/169 = 144/169$, so $\cos\theta = 12/13$
- tan=5/12; csc=13/5; sec=13/12; cot=12/5

10. Answer: r=5; sin=4/5, cos=–3/5, tan=–4/3, csc=5/4, sec=–5/3, cot=–3/4
- $r = \sqrt{(9 + 16)} = \sqrt{25} = 5$; here $x = –3$, $y = 4$

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- sin=y/r=4/5; cos=x/r=–3/5; tan=y/x=–4/3; csc=5/4; sec=–5/3; cot=–3/4

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