Graphing Secant and Cosecant Functions

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Learning Objectives

- Identify the period, amplitude factor, and vertical asymptotes of secant and cosecant functions
- Sketch the graphs of transformed secant and cosecant functions
- Relate the graphs of secant and cosecant to their reciprocal sine and cosine functions

For each function, determine the period and vertical asymptotes, then sketch the graph over one full period.

1. Find the period and the equations of the vertical asymptotes for one full cycle of the function.

$$y = 3\csc\left(\frac{1}{4}x\right)$$

Answer: ________________________________

2. Find the period and the equations of the vertical asymptotes for one full cycle of the function.

$$y = \sec(2x)$$

Answer: ________________________________

3. State the period, range, and vertical asymptotes within one period starting at x = 0.

$$y = 2\sec(x)$$

Answer: ________________________________

4. Determine the period and the first two vertical asymptotes to the right of the origin.

$$y = \csc(3x)$$

Answer: ________________________________

5. Sketch one full period and identify the period and asymptotes of the function.

$$y = \frac{1}{2}\sec\left(\frac{1}{2}x\right)$$

Answer: ________________________________

6. Find the period, range, and vertical asymptotes for one cycle starting at x = 0.

$$y = 4\csc(2x)$$

Answer: ________________________________

7. Identify the period and vertical asymptotes of the function.

$$y = \sec\left(\frac{\pi}{2}x\right)$$

Answer: ________________________________

8. Find the period and the first vertical asymptote to the right of x = 0.

$$y = -\csc\left(\frac{1}{2}x\right)$$

Answer: ______________________________________

9. State the period and the two vertical asymptotes nearest to x = 0 for the function.

$$y = 3\sec(\pi x)$$

Answer: ______________________________________

10. Determine the period and equations of the vertical asymptotes for one full cycle.

$$y = \csc\left(x - \frac{\pi}{4}\right)$$

Answer: ______________________________________

Encourage students to first sketch the reciprocal sine or cosine guide curve lightly before drawing the secant or cosecant branches.

Solutions

1. Find the period and the equations of the vertical asymptotes for one full cycle of the function.

$$y = 3\csc\left(\frac{1}{4}x\right)$$

→ *For y = a csc(bx), the period is 2π divided by b.*
→ *Here b = 1/4, so period = 2π ÷ (1/4) = 8π.*
→ *Vertical asymptotes occur where sin((1/4)x) = 0, that is x = 0, 4π, 8π.*

Answer: $\text{Period} = 8\pi;\ \text{Asymptotes: } x = 0,\ 4\pi,\ 8\pi$

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2. Find the period and the equations of the vertical asymptotes for one full cycle of the function.

$$y = \sec(2x)$$

→ *For y = sec(bx), the period is 2π divided by b.*
→ *Here b = 2, so period = 2π ÷ 2 = π.*
→ *Vertical asymptotes occur where cos(2x) = 0, that is 2x = π/2 + kπ, giving x = π/4, 3π/4.*

Answer: $\text{Period} = \pi;\ \text{Asymptotes: } x = \frac{\pi}{4},\ \frac{3\pi}{4}$

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3. State the period, range, and vertical asymptotes within one period starting at x = 0.

$$y = 2\sec(x)$$

→ *The coefficient 2 stretches the graph vertically by a factor of 2.*
→ *Period of sec(x) is 2π, unchanged by vertical stretch.*
→ *Range becomes y ≤ -2 or y ≥ 2; asymptotes occur where cos(x) = 0.*

Answer: $\text{Period} = 2\pi;\ \text{Range: } (-\infty,\ -2] \cup [2, \infty);\ x = \frac{\pi}{2},\ \frac{3\pi}{2}$

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4. Determine the period and the first two vertical asymptotes to the right of the origin.

$$y = \csc(3x)$$

→ *For csc(bx), period = 2π/b = 2π/3.*
→ *Asymptotes occur where sin(3x) = 0, so 3x = kπ.*
→ *First two positive asymptotes: x = π/3 and x = 2π/3.*

Answer: $\text{Period} = \frac{2\pi}{3};\ x = \frac{\pi}{3},\ \frac{2\pi}{3}$

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5. Sketch one full period and identify the period and asymptotes of the function.

$y = \frac{1}{2}\sec\left(\frac{1}{2}x\right)$

→ *Period = 2π divided by (1/2), which equals 4π.*
→ *The factor 1/2 in front compresses the curve vertically.*
→ *Asymptotes occur where cos((1/2)x) = 0, so (1/2)x = π/2 + kπ, giving x = π, 3π.*

Answer: $\text{Period} = 4\pi;\ \text{Asymptotes: } x = \pi,\ 3\pi$

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6. Find the period, range, and vertical asymptotes for one cycle starting at x = 0.

$y = 4\csc(2x)$

→ *Period = 2π divided by 2 = π.*
→ *Coefficient 4 stretches range to y ≤ -4 or y ≥ 4.*
→ *Asymptotes where sin(2x) = 0, so 2x = kπ, giving x = 0, π/2, π.*

Answer: $\text{Period} = \pi;\ \text{Range: } (-\infty, -4] \cup [4, \infty);\ x = 0,\ \frac{\pi}{2},\ \pi$

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7. Identify the period and vertical asymptotes of the function.

$y = \sec\left(\frac{\pi}{2}x\right)$

→ *Period = 2π divided by (π/2) = 4.*
→ *Asymptotes occur where cos((π/2)x) = 0.*
→ *Solve (π/2)x = π/2 + kπ to get x = 1, 3, ….*

Answer: $\text{Period} = 4;\ \text{Asymptotes: } x = 1,\ 3$

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8. Find the period and the first vertical asymptote to the right of x = 0.

$y = -\csc\left(\frac{1}{2}x\right)$

→ *Period = 2π divided by (1/2) = 4π.*
→ *The negative sign reflects the graph across the x-axis.*
→ *First asymptote where sin((1/2)x) = 0 with x > 0 is x = 2π.*

Answer: $\text{Period} = 4\pi;\ x = 2\pi$

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9. State the period and the two vertical asymptotes nearest to x = 0 for the function.

$y = 3\sec(\pi x)$

→ *Period = 2π divided by π = 2.*
→ *Asymptotes where cos(πx) = 0, so πx = π/2 + kπ.*
→ *Solving gives x = 1/2 + k, nearest values to 0 are 1/2 and -1/2.*

Answer: $\text{Period} = 2;\ x = \frac{1}{2},\ -\frac{1}{2}$

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10. Determine the period and equations of the vertical asymptotes for one full cycle.

$y = \csc\left(x - \frac{\pi}{4}\right)$

→ *The coefficient of x is 1, so period = 2π.*
→ *Phase shift of π/4 to the right shifts all asymptotes.*
→ *Asymptotes where sin(x - π/4) = 0, giving x = π/4 + kπ.*

Answer: $\text{Period} = 2\pi;\ x = \frac{\pi}{4},\ \frac{5\pi}{4},\ \frac{9\pi}{4}$