Graphing Cosecant and Secant Functions

Trigonometry Worksheet · Grade 11–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Identify the amplitude and period of cosecant and secant functions
- Graph cosecant and secant functions using their reciprocal sine and cosine guides
- Determine vertical asymptotes and key features of cosecant and secant graphs

Problems

1. State the reciprocal relationship: the cosecant function is the inverse (reciprocal) of which trig function?

$$y = \csc x = \frac{1}{\sin x}$$

2. Find the amplitude and period of the sine function that serves as the guide for graphing the following cosecant function.

$$y = 2\csc x$$

3. Find the amplitude and period of the cosine function that serves as the guide for graphing the following secant function.

$$y = \sec 2x$$

4. List the x-values of the vertical asymptotes for one full period of the following cosecant function over the interval 0 to 8π.

$$y = 3\csc\left(\frac{1}{4}x\right)$$

5. Find the amplitude and period of the following cosecant function, then identify the maximum and minimum values it reaches.

$$y = 4\csc x$$

Scan to watch

6. Find the period of the following secant function and state the equations of the vertical asymptotes over the interval from negative π to π.

$$y = \sec\left(\frac{1}{2}x\right)$$

7. Sketch the key features (amplitude, period, asymptotes, and vertex points) for the following cosecant function. Identify the coordinates of the vertex points for one full period.

$$y = 3\csc\left(\frac{1}{4}x\right)$$

8. Determine the amplitude, period, and vertical asymptotes over one period (0 to 2π) for the following secant function, then identify the two vertex coordinates.

$$y = -2\sec x$$

9. A cosecant function has a period of 6π and an amplitude of 5. Write its equation, then list the vertical asymptote equations for the interval from 0 to 6π.

$$y = 5\csc\left(\frac{\pi}{3} \cdot x\right)$$

10. Graph and compare the two functions below. State the period, amplitude, asymptotes over one period, and vertex coordinates for each function, then describe how the graphs differ from each other.

$$y = 2\sec\left(\frac{1}{2}x\right) \quad \text{and} \quad y = 2\csc\left(\frac{1}{2}x\right)$$

Graphing Cosecant and Secant Functions — Answer Key

Trigonometry Worksheet · Grade 11–12

Answer Key

1. Answer: Sine (sin x)

- The cosecant function is defined as $\csc x = 1/\sin x$.
- Therefore, cosecant is the reciprocal of the sine function.

2. Answer: Amplitude = 2, Period = 2π

- The guide function is $y = 2 \sin x$.
- Amplitude = $|a| = |2| = 2$.
- Period = $2\pi / B = 2\pi / 1 = 2\pi$.

3. Answer: Amplitude = 1, Period = π

- The guide function is $y = \cos 2x$.
- Amplitude = $|a| = |1| = 1$.
- Period = $2\pi / B = 2\pi / 2 = \pi$.

4. Answer: $x = 0$, $x = 4\pi$, $x = 8\pi$

- The guide function is $y = 3 \sin(1/4\ x)$ with period = $2\pi / (1/4) = 8\pi$.
- Asymptotes occur at the x-intercepts of the guide sine function.
- Sine x-intercepts over $[0, 8\pi]$: $x = 0, 4\pi, 8\pi$.
- These become the vertical asymptotes of the cosecant function.

5. Answer: Amplitude = 4, Period = 2π, Max branch vertex at 4, Min branch vertex at –4

- Guide function: $y = 4 \sin x$ with amplitude = 4 and period = 2π.
- The cosecant branches open away from the x-axis at the peaks and troughs of sine.
- Maximum vertex of the lower branch: $y = 4$ (at $\sin x = 1$).
- Minimum vertex of the upper branch: $y = -4$ (at $\sin x = -1$).

6. Answer: Period = 4π; Asymptotes: $x = -\pi$, $x = \pi$

- Guide function: $y = \cos(1/2\ x)$ with $B = 1/2$.
- Period = $2\pi / (1/2) = 4\pi$.
- Cosine equals zero (giving asymptotes for secant) at $x = \pi/2 + n\pi$ for the guide, but scaled: $x = \pi + 2n\pi$.
- Over $[-\pi, \pi]$: asymptotes at $x = -\pi$ and $x = \pi$.

7. Answer: Vertices at $(2\pi, 3)$ and $(6\pi, -3)$; asymptotes at $x = 0, 4\pi, 8\pi$

- Guide function: $y = 3 \sin(1/4\ x)$, period = 8π.
- Divide period into 4 equal parts: $0, 2\pi, 4\pi, 6\pi, 8\pi$.
- Sine maximum at $x = 2\pi \rightarrow$ cosecant vertex $(2\pi, 3)$.
- Sine minimum at $x = 6\pi \rightarrow$ cosecant vertex $(6\pi, -3)$.

Scan to watch

- Asymptotes at x-intercepts of sine: $x = 0, 4\pi, 8\pi$.

8. Answer: Amplitude = 2, Period = 2π, Asymptotes: x = π/2 and x = 3π/2; Vertices at (0, –2) and (π, 2)

- Guide function: $y = -2 \cos x$, amplitude = 2, period = 2π.
- Cosine = 0 at $x = \pi/2$ and $x = 3\pi/2$ → vertical asymptotes.
- Cosine maximum (+1) at $x = 0$: $y = -2(1) = -2$ → vertex $(0, -2)$.
- Cosine minimum (–1) at $x = \pi$: $y = -2(-1) = 2$ → vertex $(\pi, 2)$.
- The negative flips the secant branches compared to the standard graph.

9. Answer: y = 5 csc(x/3); Asymptotes: x = 0, 3π, 6π

- Period = $2\pi / B = 6\pi$ → $B = 2\pi / 6\pi = 1/3$.
- Amplitude = 5, so the equation is $y = 5 \csc(1/3\ x)$.
- Guide function: $y = 5 \sin(1/3\ x)$, x-intercepts at $x = 0, 3\pi, 6\pi$.
- Vertical asymptotes at $x = 0$, $x = 3\pi$, and $x = 6\pi$.

10. Answer: Both: amplitude = 2, period = 4π. Secant asymptotes: x = π, 3π; vertices (0,2),(2π,–2). Cosecant asymptotes: x = 0, 2π, 4π; vertices (π,2),(3π,–2). The graphs are horizontal shifts of each other by π/2.

- For $y = 2 \sec(1/2\ x)$: guide is $y = 2 \cos(1/2\ x)$, period = 4π, amplitude = 2.
- Cosine = 0 at $x = \pi$ and $x = 3\pi$ → asymptotes at $x = \pi, 3\pi$.
- Cosine vertices: $(0, 2)$ and $(2\pi, -2)$ → secant branch vertices.
- For $y = 2 \csc(1/2\ x)$: guide is $y = 2 \sin(1/2\ x)$, period = 4π, amplitude = 2.
- Sine = 0 at $x = 0, 2\pi, 4\pi$ → asymptotes there.
- Sine vertices: $(\pi, 2)$ and $(3\pi, -2)$ → cosecant branch vertices.
- Comparing both: the cosecant graph is the secant graph shifted $\pi/2$ to the right (or secant is csc shifted left by $\pi/2$), since $\sin x = \cos(x - \pi/2)$.

Scan to watch