Proving Trigonometric Identities

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Learning Objectives

- Apply fundamental trigonometric identities (reciprocal, quotient, and Pythagorean) to simplify expressions
- Prove trigonometric identities by transforming one side to match the other
- Recognize and use co-function, even-odd, and sum/difference identities in proofs

Prove each of the following trigonometric identities by showing all algebraic steps that transform the left-hand side into the right-hand side.

1. Prove the identity:

$$\sin(\theta) \cdot \cot(\theta) = \cos(\theta)$$

Answer: ______________________

2. Prove the identity:

$$\frac{1 - \cos^2(\theta)}{\sin(\theta)} = \sin(\theta)$$

Answer: ______________________

3. Prove the identity:

$$\sec(\theta) - \sin(\theta)\tan(\theta) = \cos(\theta)$$

Answer: ______________________

4. Prove the identity:

$$\frac{1 + \tan^2(\theta)}{\sec^2(\theta)} = 1$$

Answer: ______________________

5. Prove the identity:

$$\frac{\cos(\theta)}{1 - \sin(\theta)} = \frac{1 + \sin(\theta)}{\cos(\theta)}$$

Answer: ______________________

6. Prove the identity:

$$\tan(\theta) + \cot(\theta) = \sec(\theta)\csc(\theta)$$

Answer: ______________________

7. Prove the identity:

$$\frac{\sin(\theta)}{1+\cos(\theta)}+\frac{1+\cos(\theta)}{\sin(\theta)}=2\csc(\theta)$$

Answer: ______________________________________

8. Prove the identity:

$$(1-\cos(\theta))(1+\cos(\theta))=\sin^2(\theta)$$

Answer: ______________________________________

9. Prove the identity:

$$\frac{\sec^2(\theta)-1}{\sec^2(\theta)}=\sin^2(\theta)$$

Answer: ______________________________________

10. Prove the identity:

$$\csc(\theta)-\sin(\theta)=\cos(\theta)\cot(\theta)$$

Answer: ______________________________________

Encourage students to work on the more complicated side first and to convert all functions into sines and cosines when stuck.

Solutions

1. Prove the identity:

$\sin(\theta) \cdot \cot(\theta) = \cos(\theta)$

→ *Rewrite cot(θ) as cos(θ)/sin(θ).*
→ *Multiply sin(θ) by cos(θ)/sin(θ) to get sin(θ)cos(θ)/sin(θ).*
→ *Cancel sin(θ) in numerator and denominator to obtain cos(θ).*
→ *Both sides equal cos(θ), so the identity is proven.*

Answer: $\cos(\theta) = \cos(\theta)$

2. Prove the identity:

$\frac{1 - \cos^2(\theta)}{\sin(\theta)} = \sin(\theta)$

→ *Use the Pythagorean identity: 1 – $\cos^2$(θ) = $\sin^2$(θ).*
→ *Substitute into the numerator to get $\sin^2$(θ)/sin(θ).*
→ *Simplify by canceling one factor of sin(θ).*
→ *The left side reduces to sin(θ), matching the right side.*

Answer: $\sin(\theta) = \sin(\theta)$

3. Prove the identity:

$\sec(\theta) - \sin(\theta)\tan(\theta) = \cos(\theta)$

→ *Rewrite sec(θ) as 1/cos(θ) and tan(θ) as sin(θ)/cos(θ).*
→ *The expression becomes 1/cos(θ) – $\sin^2$(θ)/cos(θ).*
→ *Combine fractions to get (1 – $\sin^2$(θ))/cos(θ).*
→ *Apply the Pythagorean identity: 1 – $\sin^2$(θ) = $\cos^2$(θ).*
→ *Simplify $\cos^2$(θ)/cos(θ) to cos(θ).*

Answer: $\cos(\theta) = \cos(\theta)$

4. Prove the identity:

$\frac{1 + \tan^2(\theta)}{\sec^2(\theta)} = 1$

→ *Recall the Pythagorean identity: 1 + $\tan^2$(θ) = $\sec^2$(θ).*
→ *Substitute $\sec^2$(θ) into the numerator.*
→ *The expression becomes $\sec^2$(θ)/$\sec^2$(θ).*
→ *This simplifies to 1, matching the right side.*

Answer: $1 = 1$

5. Prove the identity:

$$\frac{\cos(\theta)}{1-\sin(\theta)} = \frac{1+\sin(\theta)}{\cos(\theta)}$$

→ *Multiply the numerator and denominator of the left side by $(1 + \sin(\theta))$.*
→ *The numerator becomes $\cos(\theta)(1 + \sin(\theta))$.*
→ *The denominator becomes $1 - \sin^2(\theta)$, which equals $\cos^2(\theta)$.*
→ *Simplify $\cos(\theta)(1 + \sin(\theta))/\cos^2(\theta)$ by canceling one $\cos(\theta)$.*
→ *Result is $(1 + \sin(\theta))/\cos(\theta)$, matching the right side.*

Answer: $$\frac{1+\sin(\theta)}{\cos(\theta)} = \frac{1+\sin(\theta)}{\cos(\theta)}$$

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6. Prove the identity:

$$\tan(\theta) + \cot(\theta) = \sec(\theta)\csc(\theta)$$

→ *Rewrite $\tan(\theta)$ as $\sin(\theta)/\cos(\theta)$ and $\cot(\theta)$ as $\cos(\theta)/\sin(\theta)$.*
→ *Find a common denominator: $\sin(\theta)\cos(\theta)$.*
→ *Combine: $(\sin^2(\theta) + \cos^2(\theta))/(\sin(\theta)\cos(\theta))$.*
→ *Apply Pythagorean identity: $\sin^2(\theta) + \cos^2(\theta) = 1$.*
→ *The expression becomes $1/(\sin(\theta)\cos(\theta)) = \csc(\theta)\cdot\sec(\theta)$.*

Answer: $$\sec(\theta)\csc(\theta) = \sec(\theta)\csc(\theta)$$

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7. Prove the identity:

$$\frac{\sin(\theta)}{1+\cos(\theta)} + \frac{1+\cos(\theta)}{\sin(\theta)} = 2\csc(\theta)$$

→ *Find a common denominator: $\sin(\theta)(1 + \cos(\theta))$.*
→ *Combine numerators: $\sin^2(\theta) + (1 + \cos(\theta))^2$.*
→ *Expand $(1 + \cos(\theta))^2 = 1 + 2\cos(\theta) + \cos^2(\theta)$.*
→ *Add: $\sin^2(\theta) + 1 + 2\cos(\theta) + \cos^2(\theta) = 2 + 2\cos(\theta) = 2(1 + \cos(\theta))$.*
→ *Divide: $2(1 + \cos(\theta))/[\sin(\theta)(1 + \cos(\theta))] = 2/\sin(\theta) = 2\csc(\theta)$.*

Answer: $$2\csc(\theta) = 2\csc(\theta)$$

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8. Prove the identity:

$$(1-\cos(\theta))(1+\cos(\theta)) = \sin^2(\theta)$$

→ *Expand the product on the left as a difference of squares: $1 - \cos^2(\theta)$.*
→ *Apply the Pythagorean identity: $1 - \cos^2(\theta) = \sin^2(\theta)$.*
→ *The left side now equals $\sin^2(\theta)$, matching the right side.*

Answer: $$\sin^2(\theta) = \sin^2(\theta)$$

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9. Prove the identity:

$$\frac{\sec^2(\theta)-1}{\sec^2(\theta)} = \sin^2(\theta)$$

→ *Split the fraction: $\sec^2(\theta)/\sec^2(\theta) - 1/\sec^2(\theta)$.*
→ *Simplify to $1 - \cos^2(\theta)$ since $1/\sec^2(\theta) = \cos^2(\theta)$.*
→ *Apply Pythagorean identity: $1 - \cos^2(\theta) = \sin^2(\theta)$.*
→ *The left side equals $\sin^2(\theta)$, matching the right side.*

Answer: $$\sin^2(\theta) = \sin^2(\theta)$$

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10. Prove the identity:

$\csc(\theta) - \sin(\theta) = \cos(\theta)\cot(\theta)$

→ *Rewrite csc(θ) as 1/sin(θ).*
→ *Combine: 1/sin(θ) – sin(θ) = (1 – $\sin^2$(θ))/sin(θ).*
→ *Apply Pythagorean identity: 1 – $\sin^2$(θ) = $\cos^2$(θ).*
→ *Rewrite as $\cos^2$(θ)/sin(θ) = cos(θ) · (cos(θ)/sin(θ)).*
→ *This equals cos(θ)·cot(θ), matching the right side.*

Answer: $\cos(\theta)\cot(\theta) = \cos(\theta)\cot(\theta)$