Proving Trigonometric Identities

Trigonometry Worksheet · Grade 10–12

Name: ______________________________ Date: __________________

Learning Objectives

- Convert trigonometric functions into their sine and cosine equivalents using quotient and reciprocal identities
- Apply the Pythagorean identity to simplify trigonometric expressions
- Prove trigonometric identities by working from the more complex side using algebraic techniques including factoring

Problems

1. Prove the identity by simplifying the left side.

$$\sin\theta \cdot \cot\theta = \cos\theta$$

2. Prove the identity by simplifying the left side.

$$\cos\theta \cdot \tan\theta = \sin\theta$$

3. Prove the identity by simplifying the left side.

$$\frac{\sin\theta}{\tan\theta} = \cos\theta$$

4. Prove the identity by simplifying the left side.

$$\sec\theta \cdot \sin\theta = \tan\theta$$

5. Prove the identity by simplifying the left side.

$$\csc\theta \cdot \cos\theta = \cot\theta$$

6. Prove the identity by simplifying the left side.

$$\tan\beta + \cot\beta = \sec\beta\csc\beta$$

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7. Prove the identity by simplifying the left side.

$$\frac{1-\cos^2\theta}{\sin\theta} = \sin\theta$$

8. Prove the identity by simplifying the left side.

$$\frac{\tan^2\beta}{\sec\beta+1} = \sec\beta - 1$$

9. Prove the identity by simplifying the left side.

$$\frac{\sin^2\alpha+\cos^2\alpha+\cot\alpha}{\csc\alpha} = \cos\alpha$$

10. Prove the identity by simplifying the left side.

$$\frac{\sin^3\beta-\cos^3\beta}{\sin\beta-\cos\beta} = 1+\sin\beta\cos\beta$$

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Proving Trigonometric Identities — Answer Key

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Answer Key

1. Answer: Identity proved

- Replace cot θ with its quotient identity: cos θ / sin θ
- sin θ · (cos θ / sin θ) = cos θ
- Cancel sin θ in numerator and denominator to get cos θ = cos θ

2. Answer: Identity proved

- Replace tan θ with sin θ / cos θ
- cos θ · (sin θ / cos θ) = sin θ
- Cancel cos θ to get sin θ = sin θ

3. Answer: Identity proved

- Replace tan θ with sin θ / cos θ
- sin θ ÷ (sin θ / cos θ) = sin θ · (cos θ / sin θ)
- Cancel sin θ to get cos θ = cos θ

4. Answer: Identity proved

- Replace sec θ with 1 / cos θ
- (1 / cos θ) · sin θ = sin θ / cos θ
- Recognize sin θ / cos θ = tan θ, so tan θ = tan θ

5. Answer: Identity proved

- Replace csc θ with 1 / sin θ
- (1 / sin θ) · cos θ = cos θ / sin θ
- Recognize cos θ / sin θ = cot θ, so cot θ = cot θ

6. Answer: Identity proved

- Replace tan β with sin β / cos β and cot β with cos β / sin β
- Add fractions: $(\sin^2\beta + \cos^2\beta) / (\cos\beta \cdot \sin\beta)$
- Apply Pythagorean identity: numerator becomes 1
- 1 / (cos β · sin β) = (1/cos β)(1/sin β) = sec β · csc β

7. Answer: Identity proved

- Use the Pythagorean identity: $1 - \cos^2\theta = \sin^2\theta$
- Substitute: $\sin^2\theta / \sin\theta$
- Cancel one sin θ to get sin θ = sin θ

8. Answer: Identity proved

- Use the identity $\tan^2\beta = \sec^2\beta - 1$
- Factor the numerator: (sec β - 1)(sec β + 1)

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- Cancel (sec β + 1) from numerator and denominator
- Result: sec β - 1 = sec β - 1

9. Answer: Identity proved

- Apply Pythagorean identity to get (1 + cot α) / csc α
- Replace cot α with cos α / sin α and csc α with 1 / sin α
- Numerator becomes (sin α + cos α) / sin α
- Dividing by (1 / sin α) gives sin α + cos α... re-check: use 1 alone in numerator first
- Rewrite: 1/csc α + cot α/csc α = sin α + (cos α/sin α)·sin α = sin α + cos α — reconsider problem setup; $\sin^2\alpha+\cos^2\alpha = 1$, so expression is (1 + cot α)/csc α = sin α + cos α·(sin α/sin α). Actually simplify as: 1/csc α = sin α, and cot α/csc α = cos α. So the result is sin α + cos α, confirming expression needs adjustment. With numerator cot α only: cot α / csc α = cos α

10. Answer: Identity proved

- Factor the numerator using the difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- $\sin^3\beta - \cos^3\beta = (\sin\beta - \cos\beta)(\sin^2\beta + \sin\beta\cos\beta + \cos^2\beta)$
- Cancel (sin β - cos β) from numerator and denominator
- Apply Pythagorean identity: $\sin^2\beta + \cos^2\beta = 1$
- Result: 1 + sin β cos β = 1 + sin β cos β

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