Sum and Difference Formulas for Trigonometric Functions

Trigonometry Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Express non-standard angles as sums or differences of special angles from the unit circle
- Apply sum and difference formulas for sine, cosine, and tangent to find exact values
- Simplify trigonometric expressions using sum and difference identities

Problems

1. Which two special angles from the unit circle can be added together to produce 75 degrees? List one valid combination.

2. Write out the cosine difference formula for two angles U and V.

$\cos(U - V) = ?$

3. Convert pi over 12 radians to degrees.

$\frac{\pi}{12}$ radians $= ?$

4. Find the exact value of cosine 75 degrees using the sum formula with 45 degrees plus 30 degrees.

$\cos(75°)$

5. Find the exact value of sine 75 degrees using the sum formula with 45 degrees plus 30 degrees.

$\sin(75°)$

6. Find the exact value of sine of pi over 12 using the difference formula.

Scan to watch

$\sin\left(\frac{\pi}{12}\right)$

7. Find the exact value of cosine 105 degrees using the sum or difference of special angles.

$\cos(105°)$

8. Find the exact value of tangent 15 degrees using the tangent difference formula.

$\tan(15°)$

9. Given that sin A equals 3 over 5 with A in quadrant I, and cos B equals negative 5 over 13 with B in quadrant II, find the exact value of sine of A plus B.

$\sin(A + B)$

10. Simplify the expression using the appropriate sum or difference identity, then evaluate it exactly.

$\cos(x + \pi) + \sin\left(x + \frac{\pi}{2}\right)$

Scan to watch

Sum and Difference Formulas for Trigonometric Functions — Answer Key

Trigonometry Worksheet · Grade 10–12

Answer Key

1. Answer: 45° + 30°

- Special angles on the unit circle include 0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°, 180°, etc.
- Check: 45° + 30° = 75°. Both 45° and 30° are special angles on the unit circle.
- Another valid combination is 120° – 45° = 75°.

2. Answer: cos U cos V + sin U sin V

- The cosine difference formula is: cos(U – V) = cos U cos V + sin U sin V.
- Note: Unlike the cosine sum formula which uses a minus sign between the terms, the difference formula uses a plus sign.

3. Answer: 15°

- Use the conversion factor: multiply radians by 180/π.
- \frac{\pi}{12} \times \frac{180°}{\pi} = \frac{180°}{12} = 15°.

4. Answer: (√6 – √2) / 4

- Write 75° = 45° + 30° and apply cos(U + V) = cos U cos V – sin U sin V.
- cos(45°) = √2/2, cos(30°) = √3/2, sin(45°) = √2/2, sin(30°) = 1/2.
- cos(75°) = (√2/2)(√3/2) – (√2/2)(1/2) = √6/4 – √2/4.
- Final answer: (√6 – √2)/4.

5. Answer: (√6 + √2) / 4

- Write 75° = 45° + 30° and apply sin(U + V) = sin U cos V + cos U sin V.
- sin(45°) = √2/2, cos(30°) = √3/2, cos(45°) = √2/2, sin(30°) = 1/2.
- sin(75°) = (√2/2)(√3/2) + (√2/2)(1/2) = √6/4 + √2/4.
- Final answer: (√6 + √2)/4.

6. Answer: (√6 – √2) / 4

- Convert π/12 to degrees: π/12 = 15°.
- Write 15° = 45° – 30° and apply sin(U – V) = sin U cos V – cos U sin V.
- sin(45°) = √2/2, cos(30°) = √3/2, cos(45°) = √2/2, sin(30°) = 1/2.
- sin(15°) = (√2/2)(√3/2) – (√2/2)(1/2) = √6/4 – √2/4.
- Final answer: (√6 – √2)/4.

7. Answer: (√2 – √6) / 4

- Write 105° = 60° + 45° and apply cos(U + V) = cos U cos V – sin U sin V.
- cos(60°) = 1/2, cos(45°) = √2/2, sin(60°) = √3/2, sin(45°) = √2/2.
- cos(105°) = (1/2)(√2/2) – (√3/2)(√2/2) = √2/4 – √6/4.

Scan to watch

- Final answer: $(\sqrt{2} - \sqrt{6})/4$.

8. Answer: $2 - \sqrt{3}$

- Write $15° = 45° - 30°$ and apply $\tan(U - V) = (\tan U - \tan V) / (1 + \tan U \tan V)$.
- $\tan(45°) = 1$, $\tan(30°) = \sqrt{3}/3$.
- $\tan(15°) = (1 - \sqrt{3}/3) / (1 + 1 \cdot \sqrt{3}/3) = ((3 - \sqrt{3})/3) / ((3 + \sqrt{3})/3)$.
- Simplify: $(3 - \sqrt{3})/(3 + \sqrt{3})$. Multiply numerator and denominator by $(3 - \sqrt{3})$.
- $(3 - \sqrt{3})^2 / (9 - 3) = (9 - 6\sqrt{3} + 3)/6 = (12 - 6\sqrt{3})/6 = 2 - \sqrt{3}$.

9. Answer: $-16/65$

- $\sin A = 3/5$, so $\cos A = 4/5$ (quadrant I, positive).
- $\cos B = -5/13$, so $\sin B = 12/13$ (quadrant II, sine is positive).
- Apply $\sin(A + B) = \sin A \cos B + \cos A \sin B$.
- $\sin(A + B) = (3/5)(-5/13) + (4/5)(12/13) = -15/65 + 48/65 = 33/65$.
- Final answer: 33/65.

10. Answer: 0

- Expand $\cos(x + \pi)$ using the sum formula: $\cos x \cos \pi - \sin x \sin \pi = \cos x(-1) - \sin x(0) = -\cos x$.
- Expand $\sin(x + \pi/2)$ using the sum formula: $\sin x \cos(\pi/2) + \cos x \sin(\pi/2) = \sin x(0) + \cos x(1) = \cos x$.
- Add the two results: $-\cos x + \cos x = 0$.
- Final answer: 0.

Scan to watch