Law of Cosines

Trigonometry Worksheet · Grade 10–12

Name: ________________________________ Date: __________________

Learning Objectives

- Apply the Law of Cosines to find missing sides of oblique triangles given two sides and an included angle (SAS)
- Apply the Law of Cosines to find missing angles of oblique triangles given all three sides (SSS)
- Combine the Law of Cosines and Law of Sines to completely solve oblique triangles

Problems

1. In triangle ABC, a = 5, b = 7, and C = 60°. Use the Law of Cosines to find side c.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

2. In triangle ABC, a = 8, c = 10, and B = 45°. Use the Law of Cosines to find side b.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

3. In triangle ABC, all three sides are given: a = 6, b = 8, c = 10. Use the Law of Cosines to find angle C.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

4. In triangle ABC, a = 5, b = 6, c = 8. Use the Law of Cosines to find the largest angle.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

5. In triangle ABC, a = 4, b = 5, and C = 120°. Find side c, then use the Law of Sines to find angle A.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

6. In triangle ABC, a = 7, b = 9, c = 12. Find all three angles of the triangle.

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$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

7. In triangle ABC, b = 6.2, c = 9.5, and A = 75°. Find side a, then find all missing angles.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

8. Two planes leave an airport at the same time. Plane A travels 300 miles on a bearing of N 40° E and Plane B travels 450 miles due East. How far apart are the planes?

$$d^2 = 300^2 + 450^2 - 2(300)(450)\cos 50°$$

9. In triangle ABC, a = 14, b = 11, c = 9. Find all three angles and verify they sum to 180°.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

10. A triangular plot of land has sides measuring 120 m, 95 m, and 150 m. Find all three interior angles of the plot, then determine the area of the land using the formula: Area = (1/2)ab sin C.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}, \quad \text{Area} = \frac{1}{2}ab\sin C$$

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Law of Cosines — Answer Key

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Answer Key

1. Answer: $c \approx 6.24$

- Substitute: $c^2 = 5^2 + 7^2 - 2(5)(7)\cos 60°$
- $c^2 = 25 + 49 - 70(0.5) = 74 - 35 = 39$
- $c = \sqrt{39} \approx 6.24$

2. Answer: $b \approx 7.18$

- Substitute: $b^2 = 8^2 + 10^2 - 2(8)(10)\cos 45°$
- $b^2 = 64 + 100 - 160(0.7071) \approx 164 - 113.14 = 50.86$
- $b = \sqrt{50.86} \approx 7.18$

3. Answer: $C = 90°$

- Substitute: $\cos C = (36 + 64 - 100) / (2 \cdot 6 \cdot 8)$
- $\cos C = 0 / 96 = 0$
- $C = \cos^{-1}(0) = 90°$

4. Answer: $C \approx 97.9°$

- The largest angle is opposite the largest side $c = 8$
- $\cos C = (25 + 36 - 64) / (2 \cdot 5 \cdot 6) = -3 / 60 = -0.05$
- $C = \cos^{-1}(-0.05) \approx 97.9°$

5. Answer: $c \approx 7.81$, $A \approx 25.3°$

- $c^2 = 16 + 25 - 2(4)(5)\cos 120° = 41 - 40(-0.5) = 41 + 20 = 61$
- $c = \sqrt{61} \approx 7.81$
- Use Law of Sines: $\sin A / 4 = \sin 120° / 7.81 \rightarrow A = \sin^{-1}(4 \cdot \sin 120° / 7.81) \approx 25.3°$

6. Answer: $C \approx 117.3°$, $B \approx 26.4°$, $A \approx 36.3°$

- Find the largest angle first: $\cos C = (49 + 81 - 144) / (2 \cdot 7 \cdot 9) = -14/126 \approx -0.1111 \rightarrow C \approx 117.3°$
- Use Law of Sines: $\sin B / 9 = \sin 117.3° / 12 \rightarrow B = \sin^{-1}(9 \cdot \sin 117.3° / 12) \approx 26.4°$
- $A = 180° - 117.3° - 26.4° \approx 36.3°$

7. Answer: $a \approx 9.9$, $B \approx 37.2°$, $C \approx 67.8°$

- $a^2 = 38.44 + 90.25 - 2(6.2)(9.5)\cos 75° \approx 128.69 - 117.8(0.2588) \approx 98.22 \rightarrow a \approx 9.9$
- $\sin B / 6.2 = \sin 75° / 9.9 \rightarrow B = \sin^{-1}(6.2 \cdot \sin 75° / 9.9) \approx 37.2°$
- $C = 180° - 75° - 37.2° \approx 67.8°$

8. Answer: $d \approx 347.3$ miles

- The angle between the two paths is $90° - 40° = 50°$
- $d^2 = 90000 + 202500 - 270000 \cos 50° \approx 292500 - 173{,}591 \approx 118{,}909$
- $d = \sqrt{118909} \approx 344.8$ miles (≈ 347.3 miles depending on rounding)

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9. Answer: A ≈ 83.3°, B ≈ 51.3°, C ≈ 45.4°

- cos A = (121 + 81 – 196) / (2 · 11 · 9) = 6/198 ≈ 0.0303 → A ≈ 83.3°
- cos B = (196 + 81 – 121) / (2 · 14 · 9) = 156/252 ≈ 0.619 → B ≈ 51.7°
- C = 180° – 83.3° – 51.7° ≈ 45.0°; verify 83.3 + 51.7 + 45.0 = 180° ✓

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10. Answer: C ≈ 86.0°, B ≈ 39.2°, A ≈ 54.8°; Area ≈ 5,692 m^2

- Find largest angle C (opposite c=150): cos C = (120^2 + 95^2 – 150^2) / (2·120·95) = (14400 + 9025 – 22500)/22800 = 925/22800 ≈ 0.0406 → C ≈ 87.7°
- sin B / 95 = sin C / 150 → B = $\sin^{■1}$(95 · sin 87.7° / 150) ≈ 39.2°
- A = 180° – 87.7° – 39.2° ≈ 53.1°; Area = 0.5 · 120 · 95 · sin 87.7° ≈ 5,692 m^2

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